Circles

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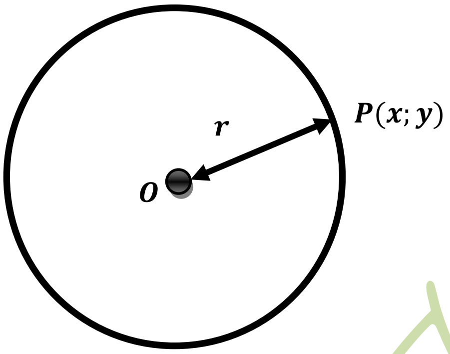
SYLLABUS (6042) REQUIREMENTS

- Find the equation of a circle
- Define a curve using parametric equations

CIRCLES

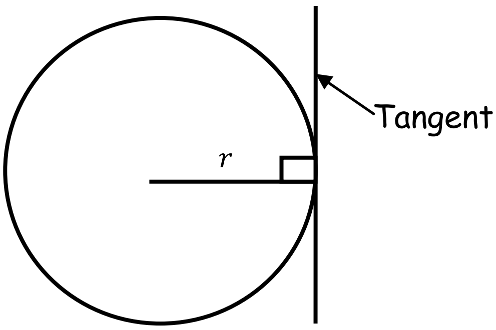


Definition

A circle is the set of all points $P\ (x, y)$ whose distance from a centre O is the distance r. Thus P is considered a point on the circle if and only if the distance from P to O equals r or simply a circle is a locus of a point P that is equidistant from another point O. The point O is called the centre of the circle and the length OP is the radius.

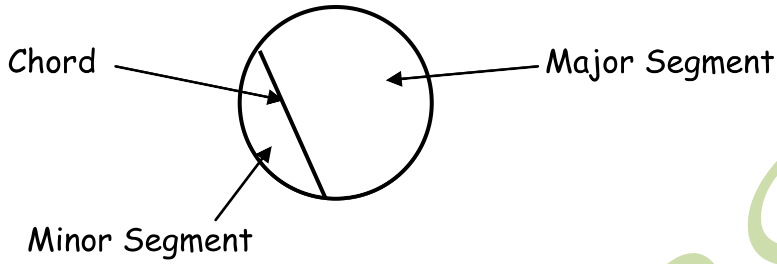
Properties of a Circle

Tangent and radius



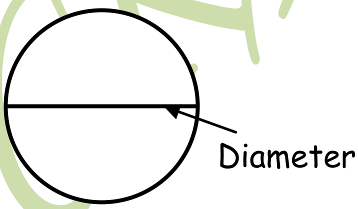
The line which touches the circle once is called the tangent and the line from the centre to the circumference is called the radius. The radius and the tangent are perpendicular.

Chord and segments



The line which touches any two points of the circle is called the chord. The chord divides the circle into two parts called the segments. The perpendicular bisector of the chord passes through the centre of the circle

Diameter



The chord which passes through the centre of the circle is called the diameter

Circle with centre $(0,0)$ and radius r

The equation of a circle with centre $(0,0)$ and radius r is given by:

$$x^2 + y^2 = r^2$$

To find the equation of the circle we need the centre and the radius.

Worked Examples

Find the equation of the circle with:

(i) centre $(0,0)$ and radius $\sqrt{3}$

(ii) centre $(0,0)$ and contains the point $(-2,4)$

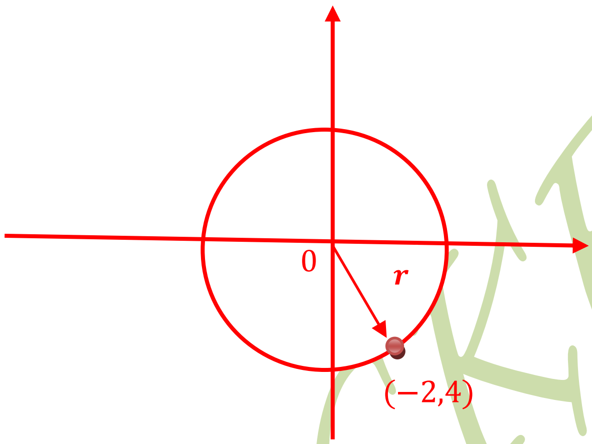
Solution

(i) $x^2 + y^2 = r^2$

$x^2 + y^2 = \left(\sqrt{3}\right)^2$

$x^2 + y^2 = 3$

(ii) centre $(0,0)$ and contains the point $(-2,4)$



$r = \sqrt{[0-(-2)]^2 + (0-4)^2} = \sqrt{4+16} = \sqrt{20}$

Now:

$x^2 + y^2 = r^2$

$x^2 + y^2 = \left(\sqrt{20}\right)^2$

$x^2 + y^2 = 20$

Alternatively, we can use the following method:

$x^2 + y^2 = r^2$

$(-2)^2 + (4)^2 = r^2$

$20 = r^2$

$\therefore x^2 + y^2 = 20$

Example

Find the coordinates of the centre and length of radius of the circles:

(i) $16x^2 + 16y^2 = 9$

(ii) $x^2 + y^2 = 19$

Suggested Solution

Find the coordinates of the centre and length of radius of the circles:

(i) $16x^2 + 16y^2 = 9$

$$x^2 + y^2 = \frac{9}{16}$$

$$x^2 + y^2 = \left(\frac{3}{4}\right)^2$$

$\therefore$ The centre is $(0,0)$ and radius is $\frac{3}{4}$.

(ii) $x^2 + y^2 = 19$

$$x^2 + y^2 = \left(\sqrt{19}\right)^2$$

$\therefore$ The centre is $(0,0)$ and radius is $\sqrt{19}$.

Circle with centre (a, b) and radius r

The equation of a circle with centre (a, b) and radius r is given by:

$$(x - a)^2 + (y - b)^2 = r^2$$

Worked Examples

Find the equation of a circle with centre $(-1, 3)$ and radius $\sqrt{2}$

Solution

$$(x - a)^2 + (y - b)^2 = r^2$$

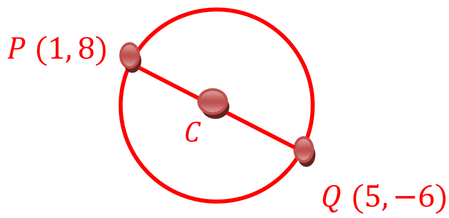
$$(x - (-1))^2 + (y - 3)^2 = \left(\sqrt{2}\right)^2$$

$$(x + 1)^2 + (y - 3)^2 = 2$$

Example

Find the equation of a circle that has points $P\ (1, 8)$ and $Q\ (5, -6)$ as the endpoints of a diameter.

Solution



Since PQ is a diameter, let's find the coordinates of the centre C.

$$C = \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$C = \left(\frac{1 + 5}{2}; \frac{8 + (-6)}{2}\right)$$

$$C = \left(\frac{6}{2}; \frac{2}{2}\right)$$

$$\therefore C = (3; 1)$$

Now finding the radius:

$$r = \frac{\sqrt{(1-5)^2 + [8-(-6)]^2}}{2} = \frac{\sqrt{(-4)^2 + (14)^2}}{2} = \frac{\sqrt{196 + 16}}{2} = \frac{\sqrt{212}}{2}$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$(x - 3)^2 + (y - 1)^2 = \left(\frac{\sqrt{212}}{2}\right)^2$$

$$(x - 3)^2 + (y - 1)^2 = \frac{212}{4}$$

$$(x - 3)^2 + (y - 1)^2 = 53$$

Example

Find the radius and centre of a circle with equation $x^2 + y^2 - 2x + 6y + 6 = 0$.

Solution

$$x^2 + y^2 - 2x + 6y + 6 = 0$$

$$x^2 - 2x + (-1)^2 - (-1)^2 + y^2 + 6y + (3)^2 - (3)^2 + 6 = 0$$

$(x-1)^2 \ -1+\ (y+3)^2-9+\ 6\ =\ 0$

$(x-1)^2+(y+3)^2-4\ =\ 0$

$(x-1)^2+\ (y+3)^2=4$

$(x-1)^2+(y+3)^2=(2)^2$

$\therefore$ The centre is $(1;-3)$ and the radius is 2.

General Equation of a Circle

The general equation of a circle is given by:

$$x^2+y^2+2gx+2fy+c=0.$$

In this case the centre is given by $(-g;-f)$ and the radius is given by $\sqrt{g^2+f^2-c}$

Proof

Remember that the equation of a circle with centre (a,b) and radius r is given by:

$$(x-a)^2+(y-b)^2=r^2$$

Now:

$$(x-a)^2+(y-b)^2=r^2 \Rightarrow x^2-2ax+a^2+y^2-2by+b^2=r^2$$
$$\Rightarrow x^2+y^2-2ax-2by+a^2+b^2-r^2=0 \qquad \text{(i)}$$

Let's equate equation (i) to the general equation $x^2+y^2+2gx+2fy+c=0$.

$\Rightarrow -2a=2g \therefore a=-g$ and also $-2b=2f \therefore b=-f$

Since we know that centre is (a,b) then when using the general equation the centre is $(-g,-f)$.

Now $a^2+b^2-r^2=c \Rightarrow (-g)^2+(-f)^2-r^2=c.$ (Since $a=-g$ and $b=-f$.)

$$\Rightarrow g^2+f^2-r^2=c$$

$$\Rightarrow g^2+f^2-c=r^2$$

$$\Rightarrow \sqrt{g^2+f^2-c}=r$$

$\therefore$ The radius is given by $\sqrt{g^2 + f^2 - c}$.

NOTE

For the equation of a circle, the coefficients of x^2 and y^2 must be the same.

Worked Example

Find the radius and centre of a circle with equation $x^2 + y^2 - 2x + 6y + 6 = 0$.

Solution

$x^2 + y^2 - 2x + 6y + 6 = 0 \quad \text{(i)}$

Let's equate equation (i) to the general equation $x^2 + y^2 + 2gx + 2fy + c = 0$.

$\Longrightarrow 2g = -2 \therefore g = -1$ and $2f = 6 \therefore f = 3$

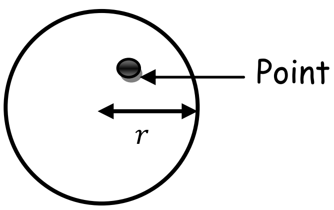
The centre is $(-g, -f) = (1; -3)$.

The radius is given by $\sqrt{g^2 + f^2 - c} = \sqrt{(1)^2 + (-3)^2 - 3} = \sqrt{10 - 6} = \sqrt{4} = 2$.

$\therefore$ The centre is $(1; -3)$ and the radius is 2.

Determining whether a point is in, on or outside the circle

Case 1: Inside the circle



A point is in the circle when either:

- The distance from the centre to the point is less that the radius
- When we substitute the point in any of the equations:
  - $x^2 + y^2 = r^2$
  - $(x-a)^2 + (y-b)^2 = r^2$
  - $x^2 + y^2 + 2gx + 2fy + c = 0$,

  yielding $LHS < RHS$.

Worked Example

Determine whether the point $(1,-2)$ lies in the circle, on the circle or outside the circle with equation $(x-1)^2 + (y+3)^2 = 4$.

Suggested Solution

Method 1

$LHS = (x-1)^2 + (y+3)^2$

Now substituting the point $(1,-2)$ to the LHS:

$$\Rightarrow (x-1)^2 + (y+3)^2 = (1-1)^2 + (-2+3)^2$$
$$= (0)^2 + (1)^2$$
$$= 0 + 1$$
$$= 1 < RHS$$

$\therefore$ The point $(1,-2)$ lies inside the circle $(x-1)^2 + (y+3)^2 = 4$.

Method 2

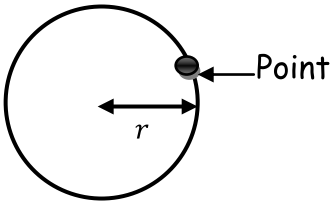
The centre is $(1;-3)$, the point is $(1,-2)$, and the radius is 2, now let's find the distance from the radius to the point.

We use the formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$$\Rightarrow \text{Distance} = \sqrt{(1-1)^2 + [-3-(-2)]^2} = \sqrt{(0)^2 + (-1)^2} = \sqrt{1} = 1 < 2\ (radius).$$

$\therefore$ The point $(1,-2)$ lies inside the circle $(x-1)^2 + (y+3)^2 = 4$.

Case 2: On the circle or lies on the circumference



A point is on the circle/lies on the circumference when either:

- The distance from the centre to the point is equal that the radius
- When we substitute the point in any of the equations:
  - $x^2 + y^2 = r^2$
  - $(x-a)^2 + (y-b)^2 = r^2$
  - $x^2 + y^2 + 2gx + 2fy + c = 0$,

  yielding $LHS = RHS$.

Worked Example

Determine whether the point $(1;-1)$ lies in the circle, on the circle or outside the circle with equation $(x-1)^2 + (y+3)^2 = 4$.

Suggested Solution

Method 1

$LHS = (x-1)^2 + (y+3)^2$

Now substituting the point $(1;-1)$ into the LHS:

$$\Rightarrow (x-1)^2 + (y+3)^2 = (1-1)^2 + (-1+3)^2$$
$$= (0)^2 + (2)^2$$
$$= 0 + 4$$
$$= 4 = RHS$$

$\therefore$ The point $(1;-1)$ lies on the circle $(x-1)^2 + (y+3)^2 = 4$.

Method 2

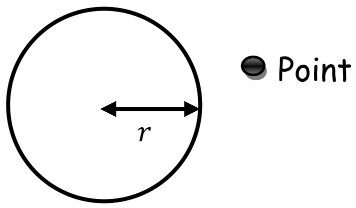
The centre is $(1;-3)$, the point is $(1;-1)$ and the radius is 2, now let's find the distance from the radius to the point.

We use the formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$\Longrightarrow Distance = \sqrt{(1-1)^2 + [-3-(-1)]^2} = \sqrt{(0)^2 + (-2)^2} = \sqrt{4} = 2 = radius.$

$\therefore$ The point $(1;-1)$ lies on the circle $(x-1)^2 + (y+3)^2 = 4$.

Case 3: Outside the circle



A point is in the circle when either:

- The distance from the centre to the point is greater that the radius
- When we substitute the point in any of the equations:
  - $x^2 + y^2 = r^2$
  - $(x-a)^2 + (y-b)^2 = r^2$
  - $x^2 + y^2 + 2gx + 2fy + c = 0$,

  yielding $LHS > RHS$.

Worked Example

Determine whether the point $(0,2)$ lies in the circle, on the circle or outside the circle with equation $(x-1)^2 + (y+3)^2 = 4$.

Suggested Solution

Method 1

$LHS = (x-1)^2 + (y+3)^2$

Now substituting the point $(0,2)$ to the LHS:

$\Longrightarrow (x-1)^2 + (y+3)^2 = (0-1)^2 + (2+3)^2$

$= (-1)^2 + (5)^2$

$= (-1)^2 + (5)^2$

$= 1 + 25$

$= 26 > RHS$

$\therefore$ The point $(0,2)$ lies outside the circle $(x-1)^2 + (y+3)^2 = 4$.

Method 2

The centre is $(1;-3)$, the point is $(0,2)$ and the radius is 2, now let's find the distance from the radius to the point.

We use the formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$\Longrightarrow Distance = \sqrt{(0-1)^2 + [2-(-3)]^2} = \sqrt{(-1)^2 + (5)^2} = \sqrt{26} > 2\ (radius)$.

$\therefore$ The point $(0;2)$, lies outside the circle $(x-1)^2 + (y+3)^2 = 4$.

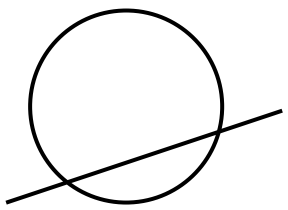
Intersection of a Circle and Line

The line of the form $y = mx + c$ and circle of the form $x^2 + y^2 + 2gx + 2fy + c = 0$ or $(x-a)^2 + (y-b)^2 = r^2$

Three cases are possible which are:

- Two points of intersection
- One point of intersection
- No point of intersection.

Case 1: Two points of intersection



When a line cuts the circle twice it is called the chord and in this case the discriminant:

$$b^2 - 4ac > 0$$

Worked Examples

ZIMSEC JUNE 2017 PAPER 1

The circle with equation $x^2 + y^2 - 4x + 6y = 12$ meets the line $3y = x + 4$ at points A and B.

Find the

(a) coordinates of the centre and radius of the circle, **[2]**

(b) length of the line AB. **[4]**

Suggested Solution

(a) $x^2 + y^2 - 4x + 6y - 12 = 0$ (i)

Let's equate equation (i) to the general equation $x^2 + y^2 + 2gx + 2fy + c = 0$.

$\Longrightarrow 2g = -4 \therefore g = -2$ and $2f = 6 \therefore f = 3$

The centre is $(-g, -f) = (2; -3)$.

The radius is given by $\sqrt{g^2 + f^2 - c} = \sqrt{(2)^2 + (-3)^2 - (-12)} = \sqrt{13 + 12} = \sqrt{25} = 5$.

$\therefore$ The centre is $(2; -3)$ and the radius is 5.

(b) To find A and B we substitute $3y = x + 4$ into $x^2 + y^2 - 4x + 6y - 12 = 0$.

$$x^2 + y^2 - 4x + 6y - 12 = 0 \Longrightarrow (3y - 4)^2 + y^2 - 4(3y - 4) + 6y - 12 = 0$$
$$\Longrightarrow (9y^2 - 24y + 16) + y^2 - 12y + 16 + 6y - 12 = 0$$
$$\Longrightarrow 10y^2 - 30y + 20 = 0$$
$$\Longrightarrow y^2 - 3y + 2 = 0$$
$$\Longrightarrow (y - 2)(y - 1) = 0$$
$$\therefore y = 1 \text{ or } 2$$

Now:

$$x = 3y - 4 \Longrightarrow x = 3(1) - 4 \text{ or } 3(2) - 4$$
$$\Longrightarrow x = 3 - 4 \text{ or } 6 - 4$$
$$\therefore x = -1 \text{ or } 2$$

$\Longrightarrow A(-1; 1)$ and $B(2; 2)$

Now to find AB we use the formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$$\Longrightarrow AB = \sqrt{[2 - (-1)]^2 + (2 - 1)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{10}.$$

$\therefore$ The length of the line $AB = \sqrt{10}$.

Example

Show that the line $y = x + 1$ is a chord to the circle with equation
$x^2 + y^2 - 4x + y - 5 = 0$.

Suggested Solution

Substituting $y = x + 1$ into $x^2 + y^2 - 4x + y - 5 = 0$.
$x^2 + y^2 - 4x + y - 5 = 0 \Longrightarrow x^2 + (x + 1)^2 - 4x + (x + 1\ ) - 5 = 0$

$$\Longrightarrow x^2 + x^2 + 2x + 1 - 4x + x + 1 - 5 = 0$$

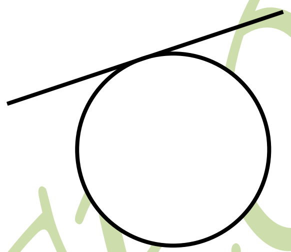
$$\Longrightarrow 2x^2 - x - 3 = 0$$

Now performing Discriminant Analysis:

$b^2 - 4ac \Longrightarrow (-1)^2 - 4(2)(-3) = 1 + 24 = 25 > 0$

Since $b^2 - 4ac > 0$ $\therefore$ the line $y = x + 1$ is a chord to the circle $x^2 + y^2 - 4x + y - 5 = 0$

Case 2 One point of intersection



When a line just touches the circle it is called the tangent and in this case the discriminant:

$$b^2 - 4ac = 0$$

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The equation of a circle $x^2 + y^2 = 4$ and the equation of its tangent is $y = 2mx + c$.

Show that $16m^2 - c + 4 = 0$. **[4]**

Suggested Solution

Since $y = 2mx + c$ is a tangent to the circle $x^2 + y^2 = 4$ then $b^2 - 4ac = 0$.

Now substituting $y = 2mx + c$ into the equation $x^2 + y^2 = 4$.

$x^2 + y^2 = 4 \Longrightarrow x^2 + (2mx + c\,)^2 = 4$

$\Longrightarrow x^2 + 4m^2x^2 + 4mcx + c^2 - 4 = 0$

$\Longrightarrow (1 + 4m^2)x^2 + 4mcx + (c^2 - 4) = 0$

Now using the discriminant $b^2 - 4ac = 0$.

$\Longrightarrow b^2 - 4ac = 0 \Longrightarrow (4mc)^2 - 4(1 + 4m^2)(c^2 - 4) = 0$

$\Longrightarrow 16m^2c^2 - 4(c^2 - 4 + 4m^2c^2 - 16m^2) = 0$

$\Longrightarrow 16m^2c^2 - 4c^2 + 16 - 16m^2c^2 + 64m^2 = 0$

$\Longrightarrow 64m^2 - 4c^2 + 16 = 0$

$\Longrightarrow 16m^2 - c^2 + 4 = 0$ (as required)

Example

Prove that the line $y = 2x - 3$ is a tangent to the circle $(x - 5)^2 + (y - 2)^2 = 5$

Suggested Solution

Substituting $y = 2x - 3$ into $(x - 5)^2 + (y - 2)^2 = 5$.

$(x - 5)^2 + (y - 2)^2 = 5 \Longrightarrow (x - 5)^2 + (2x - 3\ \ - 2)^2 = 5$

$\Longrightarrow (x - 5)^2 + (2x - 3\ \ - 2)^2 - 5 = 0$

$\Longrightarrow (x - 5)^2 + (2x - 5)^2 - 5 = 0$

$\Longrightarrow (x^2 - 10x + 25) + (4x^2 - 20x + 25) - 5 = 0$

$\Longrightarrow 5x^2 - 30x + 45 = 0$

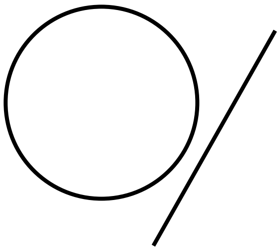
$\Longrightarrow x^2 - 6x + 9 = 0$

Now performing Discriminant Analysis:

$b^2 - 4ac \Longrightarrow (-6)^2 - 4(1)(9) = 36 - 36 = 0$

Since $b^2 - 4ac = 0 \therefore$ the line $y = 2x - 3$ is a tangent to $(x - 5)^2 + (y - 2)^2 = 5$

Case 3 No point of intersection



When a line neither touches nor cuts the circle there are no solutions and the discriminant:

$$b^2 - 4ac < 0$$

Example

Show that the line $y = x + 3$ neither touches nor cuts the circle $2x^2 + 2y^2 = 3$

Suggested Solution

Substituting the line $y = x + 3$ into the equation $2x^2 + 2y^2 = 3$:

$2x^2 + 2y^2 = 3 \Longrightarrow 2x^2 + 2(x + 3)^2 = 3$

$\Longrightarrow 2x^2 + 2(x^2 + 6x + 9) - 3 = 0$

$\Longrightarrow 2x^2 + 2x^2 + 12x + 18 - 3 = 0$

$\Longrightarrow 4x^2 + 12x + 15 = 0$

Now performing Discriminant Analysis:

$b^2 - 4ac \Longrightarrow (12)^2 - 4(4)(15) = 144 - 240 = -96 < 0$

Since $b^2 - 4ac < 0 \therefore$ the line $y = x + 3$ neither touches nor cuts the circle $2x^2 + 2y^2 = 3$.

Parametric Equations

Definition

Parametric equations are a set of equations that express a set of quantities as explicit functions of a number of independent variables knows as ***parameters***.

Case 1

Using the identity $sin^2\theta + cos^2\theta = 1$.

Worked Examples

Example 1

Find the cartesian equation, coordinates of the centre and radius length of the circle represented by the parametric equations: $x = 2 + \frac{1}{3}cos\theta$ and $y = -1 + \frac{1}{3}sin\theta$.

Solution

Let's make $cos\theta$ and $sin\theta$ the subjects of formulas.

$3(x - 2) = cos\theta$ (i) and $3(y + 1) = sin\theta$ (ii)

Now using the identity $sin^2\theta + cos^2\theta = 1$

$\Longrightarrow [3(x - 2)]^2 + [3(y + 1)]^2 = 1$

$\Longrightarrow 9(x - 2)^2 + 9(y + 1)^2 = 1$

$\Longrightarrow (x - 2)^2 + (y + 1)^2 = \frac{1}{9}$

$\therefore$ The equation of the circle is $(x - 2)^2 + (y + 1)^2 = \frac{1}{9}$

Now using $(x - a)^2 + (y - b)^2 = r^2$; to find the coordinates of the centre and length of radius.

$\Longrightarrow (x - 2)^2 + (y + 1)^2 = \left(\frac{1}{3}\right)^2$

$\therefore$ The coordinates of the centre are $(2; -1)$ and the length of radius is $\frac{1}{3}$.

Alternatively, we can write the equation of the circle as follows:

$(x - 2)^2 + (y + 1)^2 = \frac{1}{9}$

$$\Rightarrow x^2 - 4x + 4 + y^2 + 2y + 1 = \frac{1}{9}$$

$$\Rightarrow x^2 + y^2 - 4x + 2y + 4 + 1 - \frac{1}{9} = 0$$

$$\Rightarrow 9x^2 + 9y^2 - 36x + 18y + 44 = 0$$

To find the radius length we equate the above equation to $x^2 + y^2 + 2gx + 2fy + c = 0$

$\Rightarrow x^2 + y^2 - 4x + 2y + \frac{44}{9} = 0$ is compared to $x^2 + y^2 + 2gx + 2fy + c = 0$

Now comparing terms:

$-4 = 2g \Rightarrow g = -2$ and $2 = 2f \Rightarrow f = 1$

Now the centre is $(-g; -f) = (2; -1)$.

Also we use $\sqrt{g^2 + f^2 - c}$ to find the radius.

$\Rightarrow$ radius $= \sqrt{g^2 + f^2 - c}$

$$= \sqrt{(2)^2 + (-1)^2 - \frac{44}{9}}$$

$$= \sqrt{5 - \frac{44}{9}}$$

$$= \sqrt{\frac{45 - 44}{9}}$$

$$= \sqrt{\frac{1}{9}}$$

$$= \frac{1}{3}.$$

ZIMSEC NOVEMER 2015 PAPER 1

A curve has parametric equations $y = 1 + \cos\left(\frac{\pi}{3}e^{3\theta}\right)$ and $x = 2 - sin\left(\frac{\pi}{3}e^{3\theta}\right)$ for

$0 \leq \theta \leq 2\pi$.

Find the cartesian equation of the curve and describe fully what it represents geometrically.

[5]

Suggested Solution

Let's make $\cos\left(\frac{\pi}{3}e^{3\theta}\right)$ and $sin\left(\frac{\pi}{3}e^{3\theta}\right)$ the subjects of formulas.

$y - 1 = \cos\left(\frac{\pi}{3}e^{3\theta}\right)$ (i) and $x - 2 = -sin\left(\frac{\pi}{3}e^{3\theta}\right)$ (ii)

Now using the identity $sin^2\theta + cos^2\theta = 1$

$\Rightarrow \left[\cos\left(\frac{\pi}{3}e^{3\theta}\right)\ \right]^2 + \left[-sin\left(\frac{\pi}{3}e^{3\theta}\right)^2 = 1 \Rightarrow \cos^2\left(\frac{\pi}{3}e^{3\theta}\right) + sin^2\left(\frac{\pi}{3}e^{3\theta}\right) = 1\right.$

$\Rightarrow (x-2)^2 + (y-1)^2 = 1$

$\Rightarrow (x-2)^2 + (y+1)^2 = 1$

$\therefore$ It represents the equation of the circle with the coordinates of the centre are $(2; -1)$ and the length of radius is 1.

Case 2

Using the identity $x^2 + y^2 = r^2$.

Worked Example

Find the cartesian equation, coordinates of the centre and radius length of the circle represented by the parametric equations: $= \frac{t^2-4}{t^2+4}$ and $y = \frac{4t}{t^2+4}$. Write your answer in the form $x^2 + y^2 = r^2$.

Solution

Let's use the identity $x^2 + y^2 = r^2$

Now LHS:

$$x^2 + y^2 = \left(\frac{t^2 - 4}{t^2 + 4}\right)^2 + \left(\frac{4t}{t^2 + 4}\right)^2$$

$$= \frac{t^4 - 8t^2 + 16}{t^4 + 8t^2 + 16} + \frac{16t^2}{t^4 + 8t^2 + 16}$$

$$= \frac{t^4 + 16t^2 - 8t^2 + 16}{t^4 + 8t^2 + 16}$$

$$= \frac{t^4 + 8t^2 + 16}{t^4 + 8t^2 + 16}$$

$$= 1$$

$\therefore x^2 + y^2 = 1.$

Case 3

Worked Example

Find the cartesian equation of the circle represented by the parametric equations:

$x = \frac{t^2+3}{t^2+1}$ and $y = \frac{2t}{t^2+1}$.

Solution

In this case, the method used in the prior example will not yield the desired solution. Thus we can make t^2 and t the subject of formula in one equation and then we substitute into the other equation.

$x = \frac{t^2+3}{t^2+1}$ (i) and $y = \frac{2t}{t^2+1}$ (ii).

From equation (i) $x = \frac{t^2+3}{t^2+1}$

$x(t^2 + 1) = t^2 + 3 \Longrightarrow xt^2 + x = t^2 + 3$

$\Longrightarrow xt^2 - t^2 = 3 - x$

$\Longrightarrow t^2(x - 1) = 3 - x$

$\Longrightarrow t^2 = \frac{3 - x}{x - 1}$ (iii)

Also

$$t = \sqrt{\frac{3-x}{x-1}} \quad \text{(iv)}$$

Substituting equation (iv) and equation (iii) into equation (ii):

Now:

$y = \frac{2t}{t^2+1}$ (ii)

$$\Rightarrow y = \frac{2\left(\sqrt{\frac{3-x}{x-1}}\right)}{\frac{3-x}{x-1} + 1}$$

$$\Rightarrow y = \frac{2\left(\sqrt{\frac{3-x}{x-1}}\right)}{\frac{3-x+1(x-1)}{x-1}}$$

$$\Rightarrow y = \frac{2\left(\sqrt{\frac{3-x}{x-1}}\right)}{\frac{3-x+x-1}{x-1}}$$

$$\Rightarrow y = \frac{2\left(\sqrt{\frac{3-x}{x-1}}\right)}{\frac{3-1}{x-1}}$$

$$\Rightarrow y = \frac{2\left(\sqrt{\frac{3-x}{x-1}}\right)}{\frac{2}{x-1}}$$

$$\Rightarrow y = 2\left(\sqrt{\frac{3-x}{x-1}}\right) \times \frac{x-1}{2}$$

$$\Rightarrow y = \left(\sqrt{\frac{3-x}{x-1}}\right) \times (x-1)$$

Squaring both sides yields:

$$y^2 = \left[\left(\sqrt{\frac{3-x}{x-1}}\right) \times (x-1)\right]^2$$

$$\Rightarrow y^2 = \frac{3-x}{x-1} \times (x-1)^2$$

$$\Rightarrow y^2 = (3-x)(x-1)$$

$$\Rightarrow y^2 = 3x - 3 - x^2 + x$$

$$\Rightarrow y^2 = 4x - 3 - x^2$$

$\therefore$ The equation of the circle is $x^2 + y^2 - 4x + 3 = 0$.

Equation of the circle passing through three given points

Several methods can be employed but we are only going to consider just three of them.

Worked Example

Find the equation of the circle which passes through $(0;-1)$, $(1,0)$ and $(2,2)$.

Suggested Solution

Method 1: Algebraic Method.

We substitute the points $(0;-1)$, $(1,0)$ and $(2,2)$ into the general formula

$x^2 + y^2 + 2gx + 2fy + c = 0$ and solve the equations simultaneously.

At point $(0;-1)$: $x^2 + y^2 + 2gx + 2fy + c = 0$

$$\Rightarrow (0)^2 + (-1)^2 + 2g(0) + 2f(-1) + c = 0$$

$$\Rightarrow 0 + 1 + 0 - 2f + c = 0$$

$$\Rightarrow 1 - 2f + c = 0 \qquad \text{(i)}$$

At point $(1;0)$: $x^2 + y^2 + 2gx + 2fy + c = 0$

$$\Rightarrow (1)^2 + (0)^2 + 2g(1) + 2f(0) + c = 0$$

$$\Rightarrow 1 + 0 + 2g + 0 + c = 0$$

$$\Rightarrow 1 + 2g + c = 0 \qquad \text{(ii)}$$

At point $(2; 2)$: $x^2 + y^2 + 2gx + 2fy + c = 0$

$$\Rightarrow (2)^2 + (2)^2 + 2g(2) + 2f(2) + c = 0$$

$$\Rightarrow 4 + 4 + 4g + 4f + c = 0$$

$$\Rightarrow 8 + 4g + 4f + c = 0 \qquad \text{(iii)}$$

Now equation (ii) – (i):

$2g + 2f = 0 \Rightarrow g = -f$ (iv)

Also equation (ii) – (iii):

$-7 - 2g - 4f = 0$ (v)

Substituting equation (iv) into equation (v).

$-7 - 2(-f) - 4f = 0 \Rightarrow -7 + 2f - 4f = 0$

$$\Rightarrow -7 - 2f = 0$$

$$\Rightarrow 2f = -7$$

$$\therefore f = -\frac{7}{2}$$

Now $g = -f$ (iv)

$\Rightarrow g = -\left(-\frac{7}{2}\right) = \frac{7}{2}$.

Also $1 - 2f + c = 0$ (i)

$\Rightarrow 1 - 2\left(-\frac{7}{2}\right) + c = 0 \Rightarrow 1 + 7 + c = 0 \Rightarrow c = -8$.

Now:

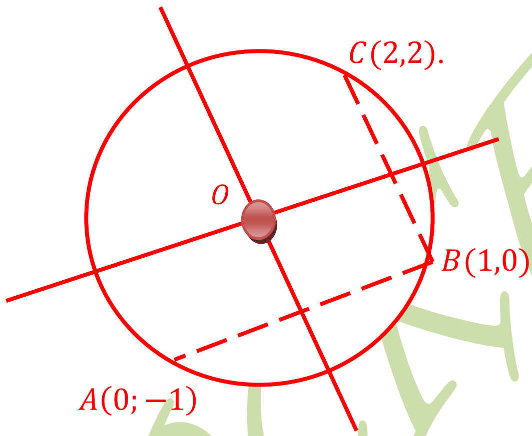
$x^2 + y^2 + 2gx + 2fy + c = 0 \Rightarrow x^2 + y^2 + 2\left(\frac{7}{2}\right)x + 2\left(-\frac{7}{2}\right)y + (-8) = 0$

$$\Rightarrow x^2 + y^2 + 7x - 7y - 8 = 0$$

$\therefore$The equation is $x^2 + y^2 + 7x - 7y - 8 = 0$.

Method 2: Geometric Method.

When using this method we have to find the midpoint between the two points and then we find the equation for the perpendicular bisector. After that we repeat the same process for the other two points. The perpendicular bisectors will intersect at the centre hence we solve them simultaneously to find the coordinates of the centre. We then find the radius and thus we obtain the equation of the circle.



Midpoint of AB $=\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$$=\left(\frac{1+0}{2};\frac{0+(-1)}{2}\right)$$

$$=\left(\frac{1}{2};-\frac{1}{2}\right)$$

Now gradient of AB $=\frac{y_2-y_1}{x_2-x_1}$

$$=\frac{0-(-1)}{1-0}$$

$$=\frac{1}{1}=1$$

$\Rightarrow$ Normal grad $=-1$

Hence equation of the perpendicular bisector is given by:

$$\frac{y-\left(-\frac{1}{2}\right)}{x-\frac{1}{2}}=-1$$

$$\Rightarrow \frac{y+\frac{1}{2}}{x-\frac{1}{2}}=-1$$

$$\Rightarrow y+\frac{1}{2}=-1\left(x-\frac{1}{2}\right)$$

$$\Rightarrow y+\frac{1}{2}=-1x+\frac{1}{2}$$

$$\Rightarrow y=-x$$

$\Rightarrow$ The equation of the perpendicular bisector is $y=-x$.

Now:

$$\text{Midpoint of BC } =\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$$

$$=\left(\frac{1+2}{2};\frac{0+2}{2}\right)$$

$$=\left(\frac{3}{2};1\right)$$

$$\text{Now gradient of BC}=\frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{2-0}{2-1}$$

$$=\frac{2}{1}=2$$

$$\Rightarrow \text{Normal grad}=-\frac{1}{2}$$

Hence equation of the perpendicular bisector is given by:

$$\frac{y-1}{x-\frac{3}{2}}=-\frac{1}{2}$$

$\Rightarrow y - 1 = -\frac{1}{2}\left(x - \frac{3}{2}\right)$

$\Rightarrow y - 1 = -\frac{1}{2}x + \frac{3}{4}$

$\Rightarrow y = -\frac{1}{2}x + \frac{3}{4} + 1$

$\Rightarrow y = -\frac{1}{2}x + \frac{7}{4}$

$\Rightarrow$ The equation of the perpendicular bisector is $y = -\frac{1}{2}x + \frac{7}{4}$.

Now we have to solve the two equations to find the centre.

$y = -\frac{1}{2}x + \frac{7}{4}$ (i)

$y = -x$ (ii)

Substituting equation (ii) into equation (i):

$y = -\frac{1}{2}x + \frac{7}{4} \Rightarrow -x = -\frac{1}{2}x + \frac{7}{4}$

$\Rightarrow -x + \frac{1}{2}x = \frac{7}{4}$

$\Rightarrow -\frac{1}{2}x = \frac{7}{4}$

$\Rightarrow x = \frac{7}{4}(-2)$

$\therefore x = -\frac{7}{2}$

$y = -x$ (ii)

$\Rightarrow y = \frac{7}{2}$

$\therefore$ The coordinates of the centre are $\left(-\frac{7}{2}; \frac{7}{2}\right)$.

Now the radius is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$\Rightarrow$ The radius $= \sqrt{\left(-\frac{7}{2} - 1\right)^2 + \left(\frac{7}{2} - 0\right)^2})$

$$= \sqrt{\left(-\frac{9}{2}\right)^2 + \left(\frac{7}{2}\right)^2}$$

$$= \sqrt{\frac{81}{4} + \frac{49}{4}}$$

$$= \sqrt{\frac{130}{4}}$$

Now the equation is given by:

$$\left[x - \left(-\frac{7}{2}\right)\right]^2 + \left(y - \frac{7}{2}\right)^2 = \left(\sqrt{\frac{130}{4}}\right)^2$$

$$\left(x + \frac{7}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{130}{4}$$

$$x^2 + 7x + \frac{49}{4} + y^2 - 7y + \frac{49}{4} = \frac{130}{4}$$

$$x^2 + y^2 + 7x - 7y + \frac{49}{4} + \frac{49}{4} - \frac{130}{4} = 0$$

$$x^2 + y^2 + 7x - 7y + \frac{98}{4} - \frac{130}{4} = 0$$

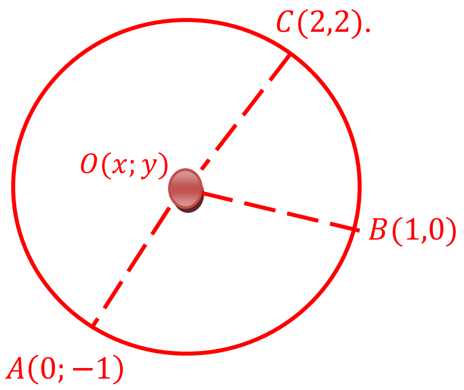
$$x^2 + y^2 + 7x - 7y + \frac{98 - 130}{4} = 0$$

$$x^2 + y^2 + 7x - 7y - \frac{32}{4} = 0$$

$$x^2 + y^2 + 7x - 7y - 8 = 0$$

Method 3: Geometric Method.

When using this method we let the coordinates of the centre be equal to $(x; y)$. We find the lengths of OA, OB and OC. We consider the fact that $|OA| = |OB| = |0C|$. We then solve the two equations simultaneously to find the coordinates of the centre. Also we find the radius and hence we have the equation of the circle.



Now:

$$OA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(x-0)^2 + [y-(-1)]^2}$$

$$= \sqrt{x^2 + (y+1)^2}$$

$$= \sqrt{x^2 + y^2 + 2y + 1}$$

Also:

$$OB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(x-1)^2 + (y-0)^2}$$

$$= \sqrt{x^2 - 2x + 1 + y^2}$$

Lastly:

$$OC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(x-2)^2 + (y-2)^2}$$

$$= \sqrt{x^2 - 4x + 4 + y^2 - 4y + 4}$$

$$= \sqrt{x^2 - 4x + y^2 - 4y + 8}$$

To find the equations:

$|0A| = |OB| \Rightarrow \sqrt{x^2 + y^2 + 2y + 1} = \sqrt{x^2 - 2x + 1 + y^2}$

$\Rightarrow \left(\sqrt{x^2 + y^2 + 2y + 1}\right)^2 = \left(\sqrt{x^2 - 2x + 1 + y^2}\right)^2$

$\Rightarrow x^2 + y^2 + 2y + 1 = x^2 - 2x + 1 + y^2$

$\Rightarrow 2y + 1 = -2x + 1$

$\Rightarrow y = -x$ (i)

Also

$|0A| = |OC| \Rightarrow \sqrt{x^2 + y^2 + 2y + 1} = \sqrt{x^2 - 4x + y^2 - 4y + 8}$

$\Rightarrow \left(\sqrt{x^2 + y^2 + 2y + 1}\right)^2 = \left(\sqrt{x^2 - 4x + y^2 - 4y + 8}\right)^2$

$\Rightarrow x^2 + y^2 + 2y + 1 = x^2 - 4x + y^2 - 4y + 8$

$\Rightarrow 2y + 1 = -4x - 4y + 8$

$\Rightarrow 2y + 4y = -4x + 8 - 1$

$\Rightarrow 6y = -4x + 7$ (ii)

Substituting equation (i) into equation (ii):

$6y = -4x + 7 \Rightarrow 6(-x) = -4x + 7$

$\Rightarrow -6x + 4x = 7$

$\Rightarrow -2x = 7$

$\therefore x = -\frac{7}{2}$

$y = -x$ (i)

$\Rightarrow y = \frac{7}{2}$

$\therefore$ The coordinates of the centre are $\left(-\frac{7}{2}; \frac{7}{2}\right)$.

Now the radius is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$\Rightarrow$ The radius $= \sqrt{\left(-\frac{7}{2}-1\right)^2+\left(\frac{7}{2}-0\right)^2})$

$$= \sqrt{\left(-\frac{9}{2}\right)^2+\left(\frac{7}{2}\right)^2}$$

$$= \sqrt{\frac{81}{4}+\frac{49}{4}}$$

$$= \sqrt{\frac{130}{4}}$$

Now the equation is given by:

$$\left[x-\left(-\frac{7}{2}\right)\right]^2+\left(y-\frac{7}{2}\right)^2=\left(\sqrt{\frac{130}{4}}\right)^2$$

$$\left(x+\frac{7}{2}\right)^2+\left(y-\frac{7}{2}\right)^2=\frac{130}{4}$$

$$x^2+7x+\frac{49}{4}+y^2-7y+\frac{49}{4}=\frac{130}{4}$$

$$x^2+y^2+7x-7y+\frac{49}{4}+\frac{49}{4}-\frac{130}{4}=0$$

$$x^2+y^2+7x-7y+\frac{98}{4}-\frac{130}{4}=0$$

$$x^2+y^2+7x-7y+\frac{98-130}{4}=0$$

$$x^2+y^2+7x-7y-\frac{32}{4}=0$$

$$x^2+y^2+7x-7y-8=0$$

Solved Past Examinations Questions

ZIMSEC NOVEMBER 2000 PAPER 1

The equation of a circle is $x^2 + y^2 + 2x - 4y = 0$.

(i) Find the coordinates of the centre of the circle, and its radius. **[3]**

(ii) Calculate the coordinates of the points of intersection of the circle and the line $y = x + 2$. **[5]**

Suggested solution

(i) $x^2 + y^2 + 2x - 4y = 0$

Let's equate equation (i) to the general equation $x^2 + y^2 + 2gx + 2fy + c = 0$.

$\Longrightarrow 2g = 2 \therefore g = 1$ and $2f = -4 \therefore f = -2$

The centre is $(-g, -f) = (-1; 2)$.

The radius is given by $\sqrt{g^2 + f^2 - c} = \sqrt{(-1)^2 + (2)^2 - 0} = \sqrt{5}$.

$\therefore$ The centre is $(-1; 2)$ and the radius is $\sqrt{5}$.

(ii) Let A and B be the points of intersection.

Now we substitute $y = x + 2$ into $x^2 + y^2 + 2x - 4y = 0$

$x^2 + y^2 + 2x - 4y = 0 \quad \Longrightarrow x^2 + (x+2)^2 + 2x - 4(x+2) = 0$

$\Longrightarrow x^2 + x^2 + 4x + 4 + 2x - 4x - 8 = 0$

$\Longrightarrow 2x^2 + 2x - 4 = 0$

$\Longrightarrow x^2 + x - 2 = 0$

$\Longrightarrow x^2 + 2x - 1x - 2 = 0$

$\Longrightarrow (x+2)(x-1) = 0$

$\therefore x = 1 \text{ or } -2$

Now:

$y = x + 2 \quad \Longrightarrow y = 1 + 2$ or $y = -2 + 2$

$\therefore y = 3$ or 0

$\Longrightarrow A(-2; 0)$ and $B(1; 3)$

ZIMSEC JUNE 2001 PAPER 1

Write down the coordinates of the centre and the radius of the circle with equation

$$(x+5)^2+(y-3)^2=36 \qquad \textbf{[2]}$$

Determine whether the origin is inside or outside this circle. **[2]**

Suggested Solution

We express the equation $(x+5)^2+(y-3)^2=36$ in the form $(x-a)^2+(y-b)^2=r^2$.
$(x+5)^2+(y-3)^2=36 \Longrightarrow (x+5)^2+(y-3)^2=6^2$

$\therefore$ The centre is $(-5;3)$ and the radius is 6.

$LHS=(x+5)^2+(y-3)^2$
Now substituting the point $(0,0)$ to the LHS:

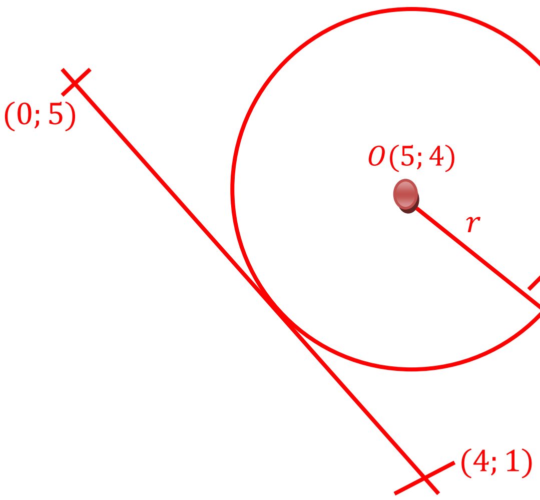
$$\begin{aligned}\Longrightarrow (x+5)^2+(y-3)^2 &= (0+5)^2+(0-3)^2\\ &= (5)^2+(-3)^2\\ &= 25+9\\ &= 34 < RHS\end{aligned}$$

$\therefore$ The point $(0,0)$ lies inside the circle $(x+5)^2+(y-3)^2=36$.

ZIMSEC NOVEMBER 2015 PAPER 1

Find the equation of the circle whose centre is $(5;4)$ and touches the line joining the points (0.5) and $(4;1)$. **[7]**

Suggested Solution



The line is the tangent to the circle and the tangent and the radius meet at A and they are perpendicular.

$$\text{Now gradient of tangent} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5-1}{0-4}$$

$$= \frac{4}{-4} = -1$$

Equation of the tangent is given by

$$\frac{y-5}{x-0} = -1 \Rightarrow y - 5 = -1(x)$$

$$\Rightarrow y = -x + 5$$

∴The equation of the tangent is $y = -x + 5$

Now finding the equation of the radius:

$\Rightarrow$ Normal grad(radius) $= 1$

Hence equation of the radius is given by:

$$\frac{y-4}{x-5} = 1$$

$\Rightarrow y - 4 = 1(x - 5)$

$\Rightarrow y - 4 = x - 5$

$\Rightarrow y = x - 5 + 4$

$\therefore y = x - 1$

Now to find A we solve the equations $y = x - 1$ (i) and $y = -x + 5$ (ii) simultaneously.

Substituting equation (i) into equation (ii):

$\Rightarrow x - 1 = -x + 5 \Rightarrow 2x = 6 \therefore x = 3.$

$y = x - 1 \Rightarrow y = 3 - 1 = 2$

$\therefore$ The coordinates of A are $(3; 2)$.

Now the radius is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$\Rightarrow$ The radius $= \sqrt{(5-3)^2 + (4-2)^2})$

$= \sqrt{(2)^2 + (2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

Now the equation is given by:

$(x-5)^2 + (y-4)^2 = \left(\sqrt{8}\right)^2$

$(x-5)^2 + (y-4)^2 = 8$ or we can simplify it further:

$x^2 - 10x + 25 + y^2 - 8y + 16 = 8$

$x^2 + y^2 - 10x - 8y + 25 + 16 - 8 = 0$

$x^2 + y^2 - 10x - 8y + 41 - 8 = 0$

$x^2 + y^2 - 10x - 8y + 33 = 0$

PAST EXAMINATIONS QUESTIONS

Ministry of Education and Cambridge NOVEMBER 1996

Find the centre and radius of the circle with equation $x^2 + y^2 - 6y = 0$. **[3]**

Find also the coordinates of the points of intersection of the line $x - 2y + 3 = 0$ and this circle. **[5]**

ZIMSEC NOVEMBER 2001 PAPER 1

A circle has centre at the point with coordinates $(-1; 2)$ and has radius 6. Find the equation of the circle, giving your answer in the form $x^2 + y^2 + ax + by + c = 0$. **[3]**

ZIMSEC NOVEMBER 2001 PAPER 1

The circle $x^2 + y^2 - 2x + 4y - 8 = 0$ has centre C and cuts the $x - axis$ at A and B.

(a) Find (i) the coordinates of C and the radius of the circle. **[3]**

(ii) the coordinates of A and B. **[2]**

(b) (i) Show that the angle $A\hat{C}B = 1.97$ radians to 3 significant figures. **[2]**

(ii) Hence find the area of that part of the circle which lies above the $x - axis$. **[3]**

ZIMSEC NOVEMBER 2003 PAPER 1

A circle touches the line $y = \frac{3}{4}x$ at the point $(4,3)$ and passes through the point $(-12; 11)$.

Find (i) the equation of the circle of the perpendicular bisector of the line passing through the points $(4; 3)$ and $(-12; 11)$, **[4]**

(ii) the equation of the circle. **[8]**

ZIMSEC NOVEMBER 2003 SPECIMEN PAPER 1

Find the centre and radius of the circle whose equation $x^2 + y^2 + 2x - y = 0$. **[3]**

Hence or otherwise, show that the gradient of the tangent at the origin is 2. **[2]**

ZIMSEC JUNE 2004 PAPER 1

The circle C has equation $x^2 + y^2 - 6x - 2y - 15 = 0$.

(i) Show that part of the line $3y = x + 5$ is a chord of the circle. **[4]**

(ii) Find the exact length of this chord. **[3]**

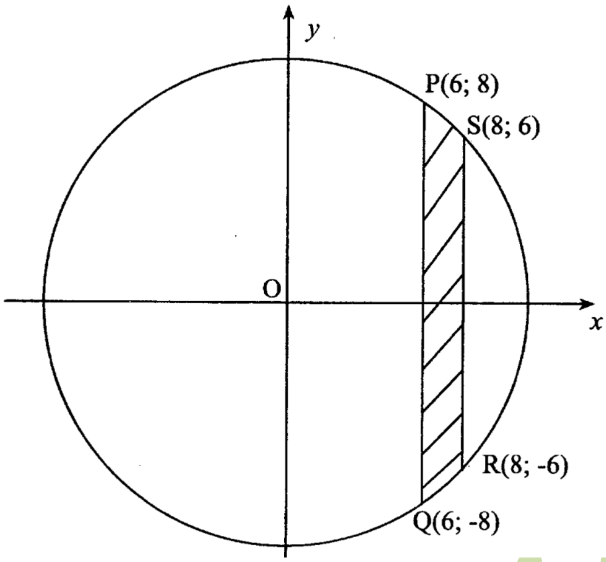
(iii) Calculate the area of the minor segment bounded by the chord and the circle correct to 2 decimal places. **[8]**

ZIMSEC NOVEMBER 2004

Find the points of intersection of the circle $x^2 + y^2 - 6x + 2y - 17 = 0$ and the line $x - y + 2 = 0$. **[5]**

Hence show that an equation of the circle which has these points as the ends of the diameter is $x^2 + y^2 - 4y - 5 = 0$. **[4]**

ZIMSEC JUNE 2005 PAPER 1



A circle centre $O(0;0)$ passes through the points $P(6;8)$, $Q(6;-8)$, $R(8;-6)$and $S(8;6)$. (See diagram)

(i) Calculate the sizes of angle POQ and angle SOR in radians correct to 3 decimal places. **[3]**

(ii) Calculate the area the shaded region $PQRS$. **[4]**

(iii) A second circle passes through the points P, O and Q. Find the coordinates of the centre of this circle. **[5]**

<u>ZIMSEC NOVEMBER 2007 PAPER 1</u>

(i) Show that the equation of the circle passing through the points $(-2;-4)$, $(3;1)$ and $(-2;0)$ is $(x-1)^2+(y-2)^2=13$. **[7]**

(ii) Deduce the equation of the diameter of the circle in (**i**) in the form $ax+by+c=0$ given that t passes through the point $(-2;0)$. **[3]**

(iii) Show that the tangent at the point $(3;1)$ is parallel to the diameter of the circle, whose equation was found in (**ii**). **[3]**

<u>ZIMSEC JUNE 2008 PAPER1</u>

A circle with centre $C(2;5)$ passes through the point $A(3;8)$.

(i) Show that the equation of the circle is $x^2+y^2-4x-10y+19=0$. **[3]**

(ii) Find the equation of the tangent at A. **[3]**

ZIMSEC NOVEMBER 2008 PAPER 1

The centre of the circle $x^2 + y^2 + 2g + 2fy + c = 0$ lies on the line $2x + y = 4$ and the circle passes through the points $(-1; 0)$ and $(-1; 4)$.

(i) Show that the coordinates of the centre of the circle are $(1; 2)$. **[3]**

(ii) Find the radius of the circle, giving your answer in exact form. **[2]**

ZIMSEC JUNE 2010 PAPER 1

Write down the equation of a circle with centre $(-3; 2)$ and radius 10. **[1]**

Show that the point $A(-2; -1)$ lies on the circle, and find the coordinates of B, the other end of the diameter through A. **[4]**

ZIMSEC NOVEBER 2010 PAPER 1

(i) Write down the equation of a circle with centre $(4; -3)$ and radius 5. **[1]**

(ii) State the condition satisfied by the point $(x; y)$ inside this circle. **[1]**

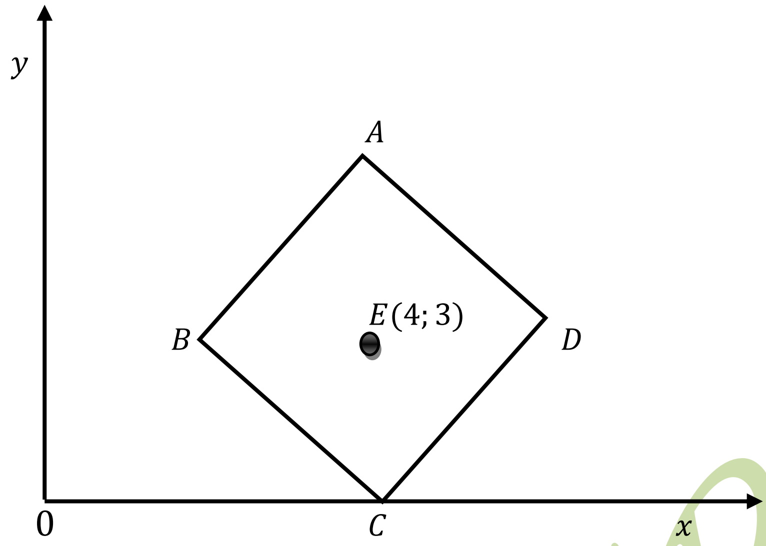
(iii) Sketch this circle and the line $2x + y = 3$ on the same diagram with the line intersecting the circle at 2 points. **[2]**

(iv) Find the range of values of x such that the point inside the circle lies on the line $2x + y = 3$. **[3]**

ZIMSEC JUNE 2012 PAPER 1

Find the equation of the circle which passes through the origin and touches the straight line $4x - 3y = 5$ at a point $(2; 1)$. **[7]**

ZIMSEC NOVEMBER 2012 PAPER 1



The diagram shows a square whose perimeter is $12\sqrt{2}\ cm$ and has centre $E(4;3)$.

(a) (i) Show that the radius of the circle which touches all the four corners of the square is $3cm$.

(ii) Find the equation of this circle giving your answer in the form

$$x^2 + y^2 + ax + by + c = 0.$$

(iii) Find the area of the minor segment enclosed between this circle and the line AD. **[8]**

It is given that the diagonal line AEC of the square s parallel to the y-axis

Write the

(i) gradients of the lines BC and BA,

(ii) equation of the tangent to the circle at point A. **[3]**

ZIMSEC JUNE 2013 PAPER 1

The curve C has parametric equations $y = at$, $x = \frac{a}{t}$ where $t > 0$.

(i) Write down the cartesian equation of C in terms of a. **[2]**

(ii) Given that the point P lies on C, show that the equation of the normal to C at P when $t = 2$, is $8y = 2x + 15a$. **[4]**

(iii)This normal meets C again at point $Q\left(-8a;\frac{-a}{8}\right)$. Given that PQ is the diameter of the circle, show that the equation of this circle is

$\left(x+\frac{15a}{4}\right)^2+\left(y-\frac{15a}{16}\right)^2=\frac{4913a^2}{256}$. **[4]**

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Calculate the centre and radius of the circle which passes through the points $P(0;0)$, $Q(1;7)$ and $R(7;-1)$. **[5]**

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The equation a straight line l_1 is $x+3y-33=0$. The straight line l_2 is parallel to l_1 and passes through $P(3;0)$.

(i) Find the equation of l_2. **[2]**

(ii) Show that the line joining P to $Q(6;9)$ is perpendicular to l_1. **[3]**

(iii) Given that the point $R(9;-2)$ lies on l_2, and that Q lies on l_1, find the equation of the circle passing through P, Q and R. **[3]**

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(a) Find the perpendicular distance of the point $P(-3;2)$ from the line whose equation is $3x-5y=7$. **[6]**

(b) Find the equation of a circle which passes through the point $(5;4)$ and touches the $y-axis$ at $(0;2)$. **[6]**

ZIMSEC JUNE 2015 PAPER 1

A triangle ABC is such that $AB=8\sqrt{3}cm$, $AC=16cm$ and $B\hat{A}C=30°$.

(i) Calculate the length BC. **[2]**

(ii) 1. Calculate the angle ACB.

2. Hence or otherwise find the radius of the circle passing through A, B and C. **[3]**

ZIMSEC JUNE 2017 PAPER 1

The circle with equation $x^2 + y^2 - 4x + 6y = 12$ meets the line $3y = x + 4$ at points A and B.
Find the

(a) coordinates of the centre and radius of the circle, [**2**]

(b) length of the line AB. **[4]**

(c) length of the perpendicular of the line AB from the centre of the circle, **[2]**

(d) the area of the triangle ABC where C is the centre of the circle with equation

$x^2 + y^2 - 4x + 6y = 12$. **[2]**

ZIMSEC NOVEMBER 2018 PAPER 1

The points $\boldsymbol{A}$ and $\boldsymbol{B}$ have position vectors $-4\lambda\boldsymbol{i} - 3\lambda\boldsymbol{j} + 4\lambda\boldsymbol{k}$ and $3\boldsymbol{i} + 9\boldsymbol{j} - 6\boldsymbol{k}$, respectively relatiuve to the origin O, where λ is a scalar. It is given that AB is perpendicular to OB.

(a) Calculate

(i) the value of λ,

(ii) the angle AOB. **[5]**

(b) Calculate the

(i) coordinates of the centre of the circle passing through **O**, **A** and **B**,

(ii) radius of the circle passing through **O**, **A** and **B**. **[4]**

ASANTE SANA

*******THERE IS A LIGHT AT THE END OF EVERY TUNNEL *******

CONSTRUCTIVE COMMENTS ON THE FORM OF THE PRESENTATION, INCLUDING ANY OMISSIONS OR ERRORS, ARE WELCOME.

ENJOY

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