Coordinate Geometry

Compiled by: Nyasha P. Tarakino (Trockers)

+263772978155/+263717267175

ntarakino@gmail.com

11 MARCH 2020

SYLLABUS (6042) REQUIREMENTS

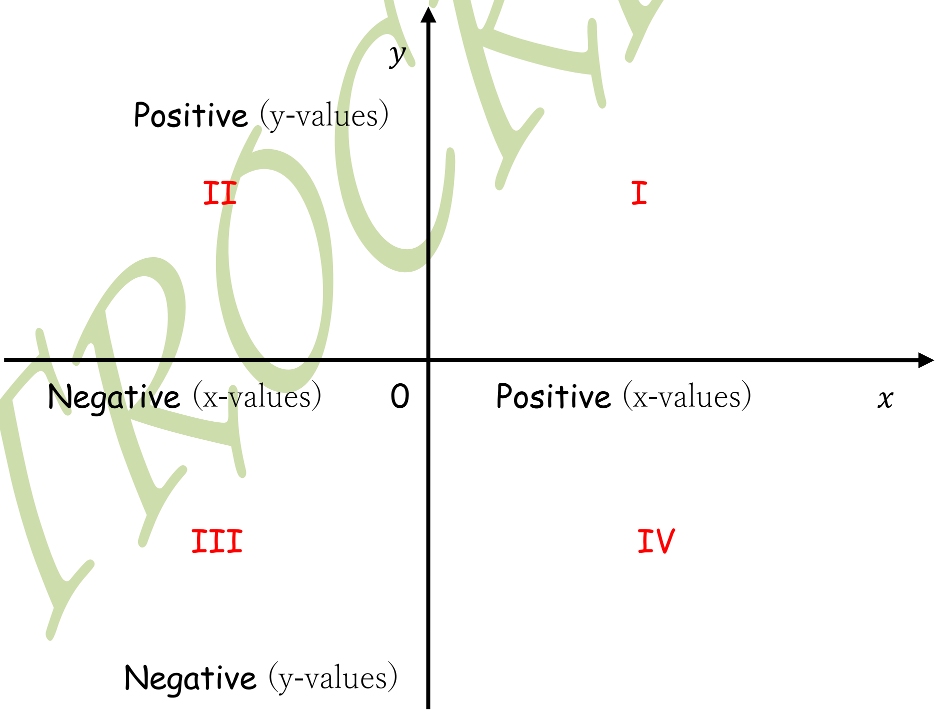
- find equations of parallel and perpendicular straight lines
- calculate the distance between two points

COORDINATE GEOMETRY

DEFINITION: It is the study of geometry using the coordinate points or the study of algebraic equations on graphs or the study of geometrically represented ordered pair of numbers

The Coordinate Plane

- Each point in a plane can be identified using an ordered pair of real numbers called the Cartesian coordinates (x, y).
- The axes are perpendicular number lines which intersect at the origin 0.
- The horizontal axis has positive numbers on the right and is called the x-axis.
- The vertical axis has positive numbers above the x-axis and is called the y-axis.
- The axes divide the plane into four quadrants, labelled I to IV.



NB: The points on the axes are not in any of the four quadrants.

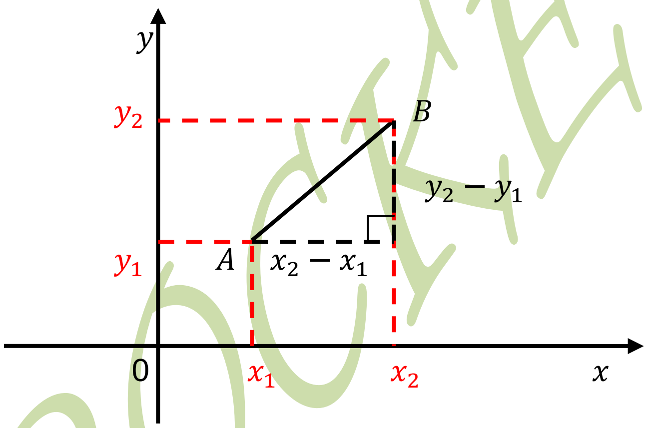
- The distance of a point from the y-axis is called its **x-coordinate**, or **abscissa**.
- The distance of a point from the x-axis is called its **y-coordinate**, or **ordinate**.
- The coordinates of a point on the x-axis are of the form $(x, 0)$
- The coordinates of a point on the y-axis are of the form $(0, y)$.

The distance between two points

- Distances in geometry are always positive, except when the points coincide.
- The distance from A to B is the same as the distance from B to A.

Distance formula

A formula used to find the distance between two points, $A(x_1, y_1)$ and $B(x_2, y_2)$.

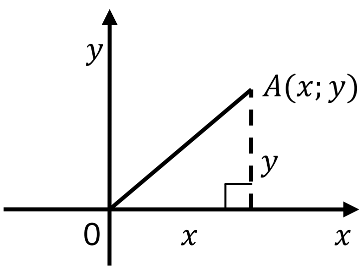


- The distance can be found using Pythagoras' theorem and it is given by:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- The distance of a point $A(x, y)$ from the origin $O(0, 0)$ is given by:

$$A = \sqrt{x^2 + y^2}$$



- Let $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ be points in $\mathbb{R}^3$then:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- The distance of a point $A(x, y, z)$ from the origin $O(0, 0,0)$ is given by:

$$A = \sqrt{x^2 + y^2 + z^2}$$

Solved Problems

Question 1

Find the distance between the points $A(1, -3)$ and $B(5, 6)$.

Suggested Solution

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(5 - 1)^2 + [6 - (-3)]^2}$$

$$= \sqrt{4^2 + 9^2}$$

$$= \sqrt{16 + 81}$$

$$= \sqrt{97}$$

Question 2

If A is $(-2, 2,3)$ and B is $(2,4, 5)$ find the length of AB.

Suggested Solution

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{[2 - (-2)]^2 + (4 - 2)^2 + (5 - 3)^2}$$

$$= \sqrt{4^2 + 2^2 + 2^2}$$

$$= \sqrt{4^2 + 2^2 + 2^2}$$

$$= \sqrt{24}$$

Question 3

Find the distance of $A(-3,4)$ from the origin.

Suggested Solution

$$A = \sqrt{x^2 + y^2}$$

$= \sqrt{(-3)^2 + 4^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= 5$

Question 4

Find the distance of $A(-1,2,7)$ from the origin.

Suggested Solution

$A = \sqrt{x^2 + y^2 + z^2}$

$= \sqrt{(-1)^2 + 2^2 + 7^2}$

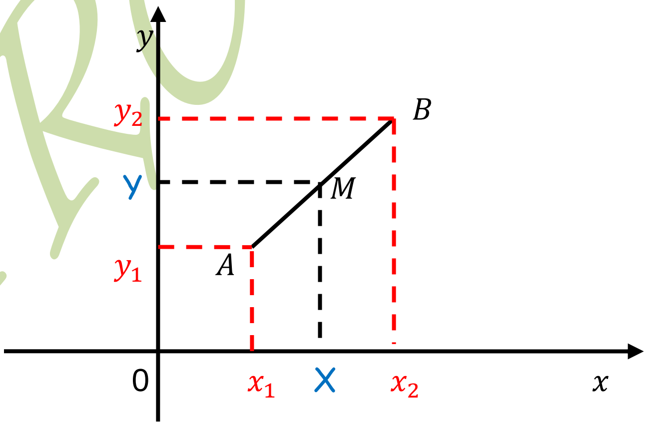
$= \sqrt{1 + 4 + 49}$

$= \sqrt{54}$

The Midpoint of an Interval

Given the points $A(x_1, y_1)$ and $B(x_2, y_2)$ then the midpoint, M, of interval AB, given by:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



NB: $X = \frac{x_1 + x_2}{2}$ and $Y = \frac{y_1 + y_2}{2}$

Solved Problems

Question 1

Find the coordinates of the midpoint of AB given that the points A is $(1, -3)$ and B is $(5, 6)$.

Suggested Solution

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{1 + 5}{2}, \frac{-3 + 6}{2}\right)$$

$$= \left(\frac{6}{2}, \frac{3}{2}\right)$$

$$= \left(3, \frac{3}{2}\right)$$

Question 2

If A is $(-2, 2, 3)$ and B is $(2, 4, 5)$, find the coordinates of the midpoint of AB.

Suggested Solution

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

$$= \left(\frac{-2 + 2}{2}, \frac{2 + 4}{2} \ \frac{3 + 5}{2}\right)$$

$$= \left(\frac{0}{2}, \frac{6}{2}, \frac{8}{2}\right)$$

$$= (0,3,4)$$

Question 3

Find the length of CD given that the point D is the midpoint of AB where A is $(2;\ 3)$, B is $(-4;\ 7)$ and C is $(3;\ 5)$.

Suggested Solution

$$D = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-4 + 2}{2}, \frac{7 + 3}{2}\right)$$

$$= \left(-\frac{2}{2}, \frac{10}{2}\right)$$

$$= (-1, 5)$$

Now:

$$CD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-3-1)^2 + (5-5)^2}$$

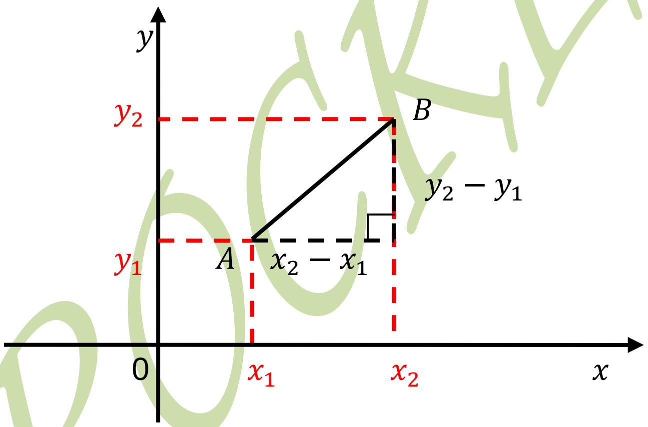
$$= \sqrt{(-4)^2 + 0}$$

$$= \sqrt{16}$$

$$= 4$$

The Gradient of a line

Definition: The gradient is a measure of the steepness of line/slope.



- We will use the pronumeral/variable m for gradient.
- The gradient of the line that passes through the points $A(x_1, y_1)$ and $B(x_2, y_2)$.is given by the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

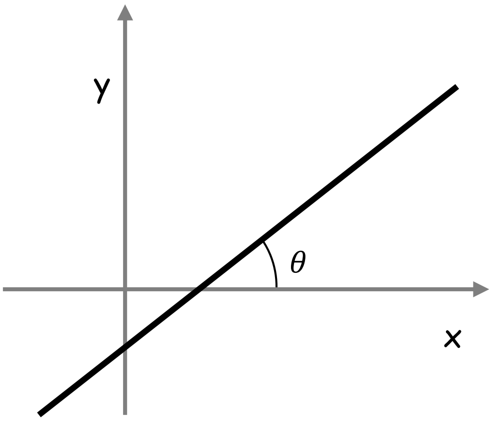
- In general, the gradient of an interval AB is given by:

$$m = \frac{rise\ (change\ in\ y)}{run\ (change\ in\ x)}$$

where rise is the change in the y-values as you move from A to B and run is the change in the x-values as you move from A to B.

Types of gradients

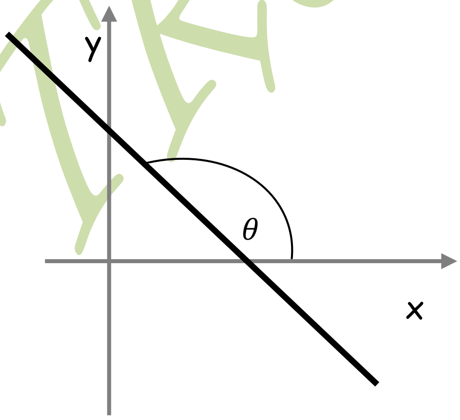
Positive Gradients (Uphill Slope)



NB: $0 < \theta < 90°$

The line going up is said to have a *positive* gradient (or slope).

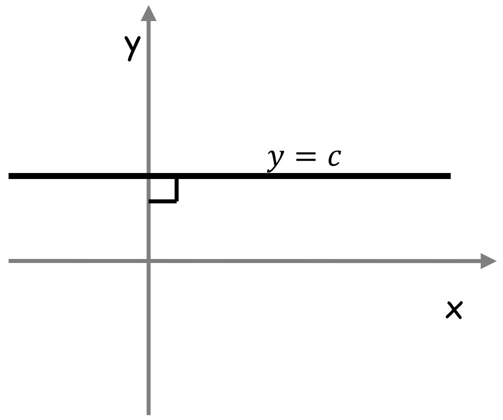
Negative Gradients (Downhill Slope)



NB: $90° < \theta < 180°$

If we move from left to right the line going down is said to have a *negative* gradient (or slope).

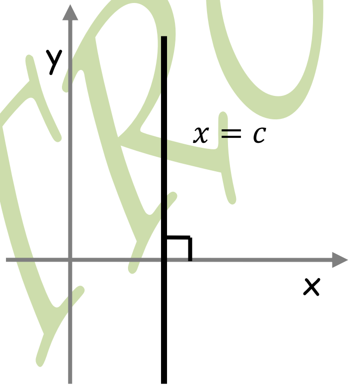
Zero Gradients (Horizontal lines)



NB: θ is not defined

If the line is horizontal (not going up or down) its gradient is zero.

No Gradients (Vertical Lines)



NB: $\theta = 90°$

If the line is vertical its gradient is not defined.

Worked Examples

Question 1

Find the gradient of the line joining the points $A(-2,3)$ and $B(2,11)$

Suggested Solution

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{11-3}{2-(-2)} \quad \text{or} \quad m = \frac{3-11}{-2-2}$$

$$= \frac{8}{4} \qquad\qquad = \frac{8}{4}$$

$$= 2 \qquad\qquad = 2$$

Question 2

The gradient of the line joining the point $A(3,-4)$ to $B(b,0)$ is 2. Find the value of b.

Suggested Solution

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\Rightarrow \frac{0-(-4)}{b-3} = 2$$

$$\Rightarrow \frac{4}{b-3} = 2$$

$$\Rightarrow 4 = 2(b-3)$$

$$\Rightarrow 4 = 2b - 6$$

$$\Rightarrow 2b = 10$$

$$\therefore b = 5$$

Question 3

Write down the value of the gradient and the coordinates of the y-axis for the line with the equation point $7x - 2y = 8$

Suggested Solution

$$7x - 2y = 8$$

$\Rightarrow 2y = 7x - 8$

$\Rightarrow y = \frac{7}{2}x - 4$

Now comparing to $y = mx + c$

$\Rightarrow m = \frac{7}{2}$ and the coordinates are $(0, -4)$

The Equation of a Line

- The general form of the equation of a line is given in the form $ax + by + c = 0$, where a, b and c are integers and $a > 0$.
- The equation of a line can also be written as $y = mx + c$, where m gives the gradient of the line and c gives the y-intercept of the line.
- Parallel lines have the same gradient

CASE 1: Given the gradient (m) and the y-intercept $(0, c)$

We use the equation $y = mx + c$

Worked Examples

Question 1

Find the equation of the line which cuts the y-axis 4 and its gradient is $\frac{3}{2}$.

Suggested Solution

$y = mx + c$

$\Rightarrow y = \frac{3}{2}x + 4$ or $2y = 3x + 8$

Question 2

Find the equation of the line which cuts the y-axis $-\frac{5}{3}$ and its gradient is 3.

Suggested Solution

$y = mx + c$

$\Rightarrow y = 3x - \frac{5}{3}$ or $3y = 9x - 5$

CASE 2: Given the gradient (m) and the point (x_1, y_1)

We use the equation $y - y_1 = m(x - x_1)$

Worked Examples

Question 1

Find the equation of the line which passes through the point $(-2,4)$ and has a gradient of 5.

Suggested Solution

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 4 = 5[x - (-2)]$

$\Rightarrow y - 4 = 5(x + 2)$

$\therefore y = 5x + 14$

Question 2

Find, in the form $ax + by + c = 0$, the equation of the line which passes through the point $(3,3)$ with a gradient of $\frac{1}{4}$.

Suggested Solution

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 3 = \frac{1}{4}(x - 3)$

$\Rightarrow 4(y-3) = x-3$

$\Rightarrow 4y - 12 = x - 3$

$\Rightarrow x - 4y - 3 + 12 = 0$

$\therefore x - 4y + 9 = 0$

CASE 3: Given two points (x_1, y_1) and (x_2, y_2)

Method 1

Steps

(i) Find the value of the gradient m, using the given points.

(ii) Find the vertical axis intercept c using the equation $y = mx + c$, by substituting the value of m and the coordinates of one of the given points.

(iii) Rewrite $y = mx + c$ replacing m and c with their numerical values.

Method 2

We can use the formula:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

where (x_1, y_1) and (x_2, y_2) are points on the line.

Worked Example

Question

Find the equation of the line which passes through the points $(-2,4)$ and $(2,-8)$.

Suggested Solution

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$\Rightarrow y - 4 = \frac{-8-4}{2-(-2)}[x-(-2)]$

$\Rightarrow y - 4 = \frac{-12}{4}(x+2)$

$\Rightarrow y - 4 = -3(x+2)$

$\therefore y = -3x - 2$

Alternatively we can use the following method:

Find the gradient

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{4-(-8)}{-2-2}$ or $m = \frac{-8-4}{2-(-2)}$

$= \frac{12}{-4}$ $= \frac{-12}{4}$

$= -3$ $= -3$

Find the y-intercept (c)

Choose any point and substitute it into $y = mx + c$

$4 = -3(-2) + c$

$\Rightarrow 4 = 6 + c$

$\therefore c = -2$

Rewrite $y = mx + c$ replacing m and c with their numerical values

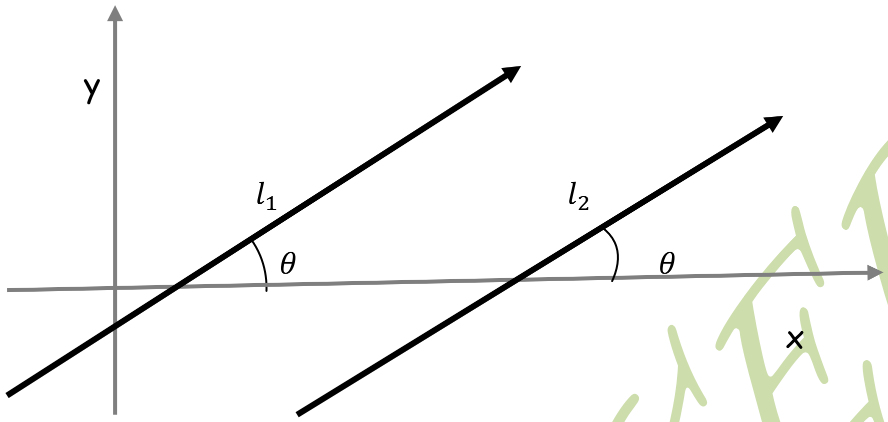
$y = mx + c$

$\Rightarrow y = -3x - 2$

Parallel and Perpendicular Lines

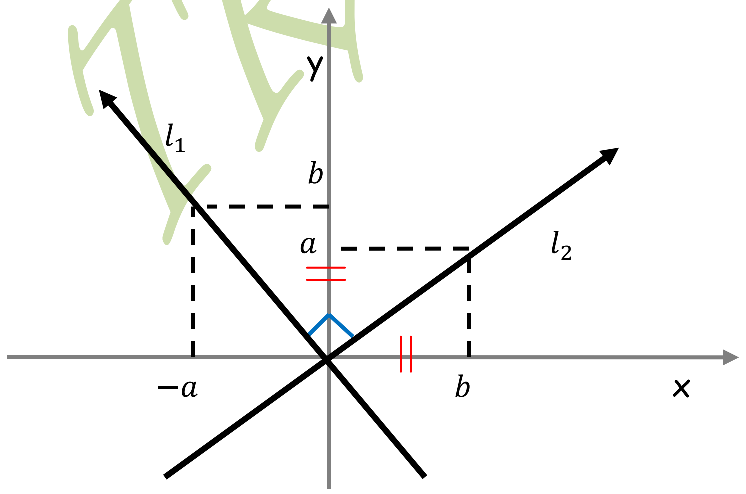
Parallel Lines

Two lines l_1 and l_2 with respective gradients of m_1 and m_2 are parallel and only if they have the same slope i.e. if $m_1 = m_2$ and also if their corresponding angles are equal.



Perpendicular Lines

Two lines l_1 and l_2 with respective gradients of m_1 and m_2 are perpendicular if and only if the product of their slopes is -1 i.e. if $m_1 m_2 = -1$ where neither m_1 nor m_2 can equal zero. In other words, the two slopes must be opposite (opposite signs) reciprocals.



$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{b - 0}{-a - 0}$$
$$= -\frac{b}{a}$$

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{a - 0}{b - 0}$$
$$= \frac{a}{b}$$

Now:

$$m_1 m_2 = -\frac{b}{a} \times \frac{a}{b}$$
$$= -1$$

$$\Rightarrow m_2 = -\frac{1}{m_1}$$

Worked Examples

Question 1

Find the equation of the line which is perpendicular to the line which passes through the points $(-2,4)$ and $(2,-8)$.

Suggested Solution

NB: For perpendicular lines:

$$m_2 = -\frac{1}{m_1}$$
$$= -\frac{1}{\left(\frac{y_2 - y_1}{x_2 - x_1}\right)}$$
$$= -\frac{x_2 - x_1}{y_2 - y_1}$$

Now:

$$y - y_1 = -\frac{x_2 - x_1}{y_2 - y_1}(x - x_1)$$

$$\Rightarrow y - 4 = -\frac{2 - (-2)}{-8 - 4}[x - (-2)]$$

$$\Rightarrow y - 4 = \frac{4}{12}(x + 2)$$

$\Rightarrow 3(y-4) = (x+2)$

$\Rightarrow 3y - 12 = x + 2$

$\therefore 3y = x + 14$

Alternatively we can use the following method:

Find the gradient

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{4-(-8)}{-2-2} \quad \text{or} \quad m_1 = \frac{-8-4}{2-(-2)}$$

$$= \frac{12}{-4} \qquad\qquad = \frac{-12}{4}$$

$$= -3 \qquad\qquad = -3$$

$$\Rightarrow m_2 = -\frac{1}{m_1}$$

$$= -\frac{1}{-3}$$

$$= \frac{1}{3}$$

Find the y-intercept (c)

Choose any point and substitute it into $y = mx + c$

$$4 = \frac{1}{3}(-2) + c$$

$$\Rightarrow 4 = -\frac{2}{3} + c$$

$$\therefore c = \frac{14}{3}$$

Rewrite $y = mx + c$ replacing m and c with their numerical values

$$y = mx + c$$

$$\Rightarrow y = \frac{1}{3}x + \frac{14}{3}$$

$$\therefore 3y = x + 14$$

Question 2

Find the equation of the line which passes through the point $(-2,4)$ and which is parallel to the line with equation $y = 5x + 11$.

Suggested Solution

NB: For parallel lines:

$$m_2 = m_1$$

$$\Rightarrow m_2 = m_1 = 5$$

Now

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 4 = 5[x - (-2)]$

$\Rightarrow y - 4 = 5(x + 2)$

$\therefore y = 5x + 14$

Follow Up Questions

Question 1

For the following pairs of points A and B, calculate:

(a) the midpoint of the line joining A to B

(b) the distance AB

(c) the gradient of the line AB

(d) the gradient of the line perpendicular to AB.

(i) $A(3,2)$ and $B(5,9)$

Answer: $\left(4;\frac{11}{2}\right)$, $\sqrt{53}$, $\frac{7}{2}$ and $-\frac{2}{7}$

(ii) $A(-3,-2)$ and $B(-5,-9)$

Answer: $\left(-4;-\frac{11}{2}\right)$, $\sqrt{53}$, $\frac{7}{2}$ and $-\frac{2}{7}$

(iii) $A(-3,-2)$ and $B(5,9)$

Answer: $\left(1;\frac{7}{2}\right)$, $\sqrt{185}$, $\frac{11}{8}$ and $-\frac{8}{11}$

(iv) $A(-3,2)$ and $B(5,-9)$

Answer: $\left(1;-\frac{7}{2}\right)$, $\sqrt{185}$, $-\frac{11}{8}$ and $\frac{8}{11}$

Question 2

The points A, B and C have coordinates $(2,1), (b,3)$ and $(5,5)$, where $b > 3$, and $ABC = 90°$.

Find:

(i) the value of b

(ii) the lengths of AB and BC

(iii) the area of triangle ABC.

Answer: $b = 6$, $AB = \sqrt{20}, BC = \sqrt{5}$; Area $= 5\text{units}^2$

Solved Past Examination Questions

Question 1

UCLES NOVEMBER 1996 PAPER 1

The line joining P, Q, R have coordinates $(3,-1)$, $(2,5)$, $(8,3)$ respectively. Find the equation of the straight line joining P to the mid-point of QR. **[3]**

Suggested Solution

Let the mid-point of $QR = M$

$$M = \left(\frac{2+8}{2};\frac{5+3}{2}\right)$$

$$= (5;4)$$

Now:

$$Gradient\ (PM) = \left(\frac{4-\{-1\}}{5-3}\right)$$

$$= \frac{5}{2}$$

Now the equation is given by:

$$\Rightarrow \frac{y-4}{x-5} = \frac{5}{2}$$

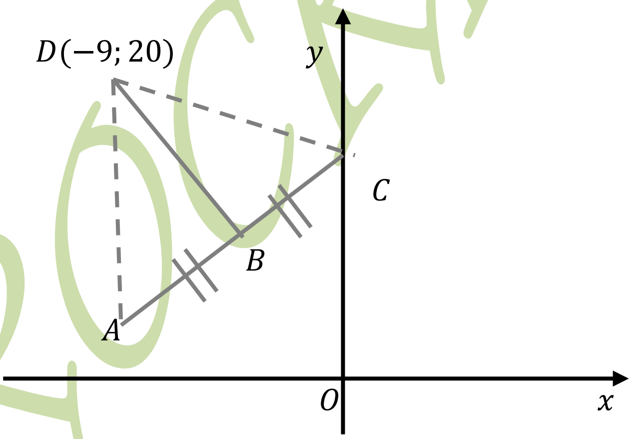
$$\Rightarrow 2(y-4) = 5(x-5)$$

$$\Rightarrow 2y - 8 = 5x - 25$$

$$\therefore 5x - 2y = 17$$

Question 2

ZIMSEC November 2019 Paper 1



The diagram above shows points A, B and C lying on the line whose equation is $3y = x + 9$. The point C lies on the y-axis and $AB = BC$. The line from $D(-9;20)$ to B is perpendicular to AC.

Calculate the

(a) co-ordinates of A and B, **[6]**

(b) exact area of triangle ADC. **[3]**

Suggested Solution

(a) At $C, x = 0$: $3y = x + 9$

$3y = 0 + 9$

$3y = 9$

$\frac{3y}{3} = \frac{9}{3}$

$y = 3$

$\therefore C$ is $(0; 3)$

Also the gradient of DB is perpendicular to the gradient AC

Now: gradient of $AC = \frac{1}{3} \Rightarrow$ gradient of $DB = -3$

Equation of DB is given by:

$\frac{y - 20}{x + 9} = -3$

$\Rightarrow y - 20 = -3(x + 9)$

$\Rightarrow y = -3x - 7$

Finding the coordinates of B

$y = -3x - 7$ (i)

$3y = x + 9$ (ii)

Substituting equation (i) into equation (ii)

$\Rightarrow 3(-3x - 7) = x + 9$

$\Rightarrow -9x - 21 = x + 9$

$\Rightarrow -10x = 9 + 21$

$\Rightarrow 10x = 30$

$\Rightarrow x = -3$

Now:

$y = -3x - 7$

$\Rightarrow y = -3(-3) - 7$

$\Rightarrow y = 9 - 7$

$\Rightarrow y = 2$

$\therefore$The coordinates of B are $(-3; 2)$

Let the coordinates of A be $(x;\ y)$

$B(-3;2)$ and $C(0;3)$

Since $AB = BC$

$$\Rightarrow -3 = \frac{x+0}{2} \Rightarrow x = -6$$

and

$$\Rightarrow 2 = \frac{y+3}{2} \Rightarrow y+3 = 4 \Rightarrow y = 1$$

∴The coordinates of A are $(-6;1)$

(b) $A(-6;1), B(-3;2), C(0;3)$ and $D(-9;20)$

Length of $DB = \sqrt{(-3+9)^2 + (2-20)^2}$

$= \sqrt{(6)^2 + (-18)^2}$

$= \sqrt{36+324}$

$= \sqrt{360}$

Length of $AC = \sqrt{(0+6)^2 + (3-1)^2}$

$= \sqrt{(6)^2 + (2)^2}$

$= \sqrt{36+4}$

$= \sqrt{40}$

Now:

$Area = \frac{1}{2}bh$

$= \frac{1}{2}(\sqrt{360})(\sqrt{40})$

$= \frac{1}{2}(\sqrt{14\,400})$

$= \frac{1}{2}(120)$

$= 60\ units^2$

Question 3

Cambridge June 2004 Paper 1

The curve $y = 9 - \frac{6}{x}$ and the line $y + x = 8$ intersect at two points. Find

(i) the coordinates of the two points, **[4]**

(ii) the equation of the perpendicular bisector of the line joining the two points. **[4]**

Suggested Solution

(i) $y = 9 - \frac{6}{x}$ (i)

$y + x = 8$ (ii)

Substituting equation (i) into equation (ii)

$\Rightarrow 8 - x = 9 - \frac{6}{x}$

$\Rightarrow x(8 - x) = 9x - 6$

$\Rightarrow 8x - x^2 = 9x - 6$

$\Rightarrow x^2 + x = 6$

$\Rightarrow x^2 + x + \left(\frac{1}{2}\right)^2 = 6 + \left(\frac{1}{2}\right)^2$

$\Rightarrow \left(x + \frac{1}{2}\right)^2 = 6 + \frac{1}{4}$

$\Rightarrow \left(x + \frac{1}{2}\right)^2 = \frac{25}{4}$

$\Rightarrow x + \frac{1}{2} = \pm\sqrt{\frac{25}{4}}$

$\Rightarrow x = -\frac{1}{2} \pm \frac{5}{2}$

$\Rightarrow x = -\frac{1}{2} + \frac{5}{2}$ or $-\frac{1}{2} - \frac{5}{2}$

$\Rightarrow x = \frac{4}{2}$ or $-\frac{6}{2}$

$\Rightarrow x = -3$ or 2

Now:

$y + x = 8$ (ii)

$y = -x + 8$

$\Rightarrow y = -(-3) + 8$ or $-2 + 8$

$\Rightarrow y = 11$ or 6

$\therefore$ The coordinates are $(-3; 11)$ and $(2; 6)$

(ii) Let the mid-point of $QR = M$

$$M = \left(\frac{-3+2}{2}; \frac{11+6}{2}\right)$$

$$= \left(-\frac{1}{2}; \frac{17}{2}\right)$$

Now:

Gradient of line $= \left(\frac{6-11}{2-\{-3\}}\right)$

$$= -\frac{5}{5}$$

$$= -1$$

$\therefore$ Gradient of perpendicular line $= 1$

Now the equation is given by:

$$\Rightarrow \frac{y - \frac{17}{2}}{x + \frac{1}{2}} = 1$$

$$\Rightarrow \frac{2y - 17}{2x + 1} = 1$$

$\Rightarrow 2y - 17 = 1(2x + 1)$

$\Rightarrow 2y - 17 = 2x + 1$

$\Rightarrow 2y = 2x + 18$

$\therefore y = x + 9$

Question 4

ZIMSEC JUNE 2005 PAPER 1

The points A and B have coordinates $(m^2; 2m)$ and $(3m^2; 6m)$ respectively.

Find the values of m for which the mid-point of AB lies on the line $y = 2x - 8$. **[3]**

Suggested Solution

Let the mid-point of AB $= M$

$$M = \left(\frac{m^2 + 3m^2}{2}; \frac{2m + 6m}{2}\right)$$

$$= (2m^2; 4m)$$

Since the mid-point of AB lies on the line $y = 2x - 8$

$\Rightarrow y = 2x - 8 \Rightarrow 4m = 2m^2 - 8$

$$\Rightarrow m^2 - 2m = 4$$

$$\Rightarrow m^2 - 2m + (-1)^2 = 4 + (-1)^2$$

$$\Rightarrow (m - 1)^2 = 5$$

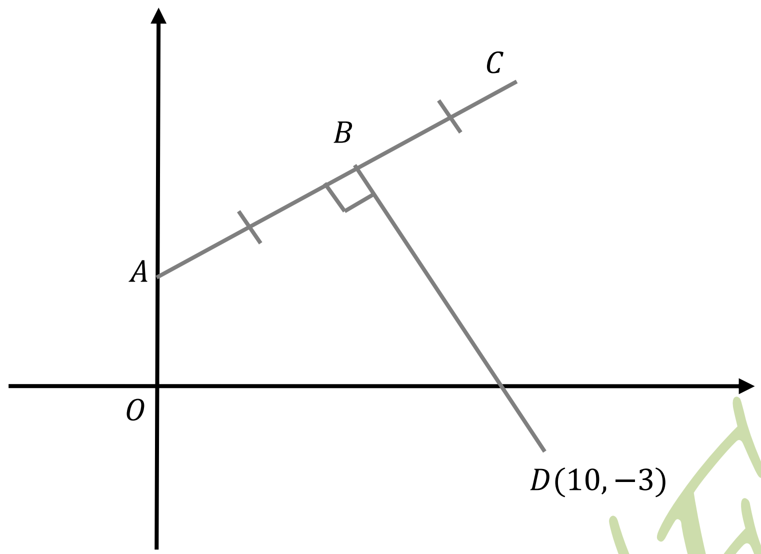
$$\Rightarrow m - 1 = \pm\sqrt{5}$$

$$\Rightarrow m = 1 \pm \sqrt{5}$$

$$\therefore m = 1 + \sqrt{5} \ \ or \ \ 1 - \sqrt{5}$$

Question 5

Cambridge June 2009 Paper 1



The diagram shows points A, B and C lying on the line $2y = x + 4$. The point A lies on the y-axis and $AB = BC$. The line from $D(10, -3)$ to B is perpendicular to AC. Calculate the coordinates of B and C. **[7]**

Suggested Solution

At $A, x = 0$: $2y = x + 4$

$2y = 0 + 4$

$2y = 4$

$\frac{2y}{2} = \frac{4}{2}$

$y = 2$

$\therefore A$ is $(0; 2)$

Also the gradient of DB is perpendicular to the gradient AC

Now: gradient of $AC = \frac{1}{2} \Rightarrow$ gradient of $DB = -2$

Equation of DB is given by:

$$\frac{y+3}{x-10} = -2$$

$\Rightarrow y + 3 = -2(x - 10)$

$\Rightarrow y = -2x + 17$

Finding the coordinates of B

$y = -2x + 17$ (i)

$2y = x + 4$ (ii)

Substituting equation (i) into equation (ii)

$\Rightarrow 2(-2x + 17) = x + 4$

$\Rightarrow -4x + 34 = x + 4$

$\Rightarrow -5x = 4 - 34$

$\Rightarrow -5x = -30$

$\Rightarrow x = 6$

Now:

$y = -2x + 17$

$\Rightarrow y = -2(6) + 17$

$\Rightarrow y = -12 + 17$

$\Rightarrow y = 5$

∴The coordinates of B are $(6; 5)$

Let the coordinates of C be $(x;\ y)$

$B(6; 5)$ and $C(0; 2)$

Since $AB = BC$

$\Rightarrow 6 = \frac{x+0}{2} x \Rightarrow x = 12$

and

$\Rightarrow 5 = \frac{y+2}{2} \Rightarrow y + 2 = 10 \Rightarrow y = 8$

∴The coordinates of C are $(12; 8)$

Questions 7

ZIMSEC JUNE 2019 PAPER 1

Find in terms of k, the equation of the perpendicular bisector of the line joining the points $A(2k;\ k)$ and $B(4k; 9k)$. **[3]**

Suggested Solution

Gradient of $AB = \frac{9k - k}{4k - 2k}$

$= \frac{8k}{2k}$

$= 4$

$\Rightarrow$ gradient of $= -\frac{1}{4}$

Let the mid-point of $AB\ = M$

$M = \left(\frac{2k+4k}{2}; \frac{k+9k}{2}\right)$

$= (3k; 5k)$

Now the equation of the perpendicular bisector is given by:

$\frac{y - 5k}{x - 3k} = -\frac{1}{4}$

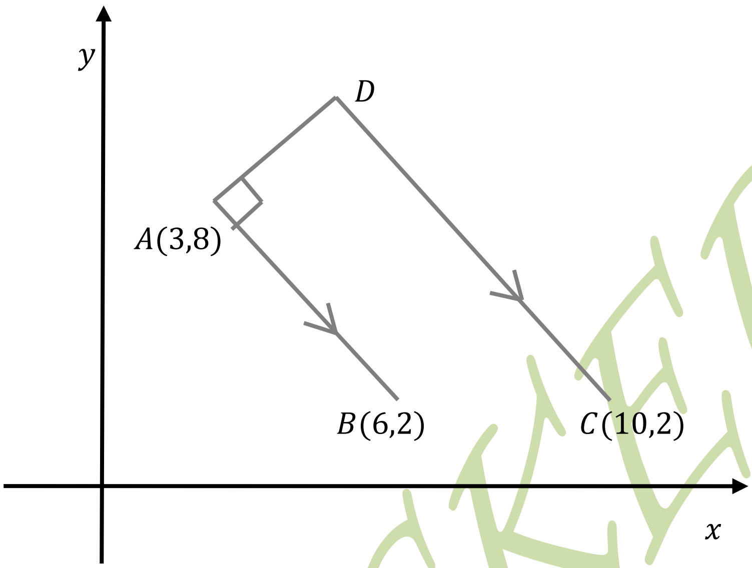
$\Rightarrow 4(y - 5k) = -1(x - 3k)$

$\Rightarrow 4y - 20k = -x + 3k$

$\Rightarrow x + 4y = 23k$

Question 8

Cambridge November 2007 Paper 1



The three points $A\ (3, 8), B\ (6, 2)$ and $C\ (10, 2)$ are shown in the diagram. The point D is such that the line DA is perpendicular to AB and DC is parallel to AB. Calculate the coordinates of D. **[7]**

Suggested Solution

Gradient of $AB = \dfrac{8-2}{3-6}$

$= -\dfrac{6}{3}$

$= -2$

Now:

Gradient of $AD\ (normal) = \dfrac{1}{2}$

Equation of AD is given by:

$$\frac{y-8}{x-3}=\frac{1}{2}$$

$\Rightarrow 2y-16=x-3$

$\Rightarrow 2y=x+13$

Also:

Gradient of CD $(parallel\ to\ AB)=-2$

Equation of CD is given by:

$$\frac{y-2}{x-10}=-2$$

$\Rightarrow y-2=-2x+20$

$\Rightarrow y=-2x+22$

Finding D:

$2y=x+13$ (i)

$y=-2x+22$ (ii)

Substituting equation (ii) into equation (i)

$\Rightarrow 2(-2x+22)=x+13$

$\Rightarrow -4x+44=x+13$

$\Rightarrow -5x=-31$

$\Rightarrow x=\frac{31}{5}$ $(or\ 6.2)$

$y=-2x+22$ (ii)

$\Rightarrow y=-2\left(\frac{31}{5}\right)+22$

$\Rightarrow y=-\frac{26}{5}+22$

$\Rightarrow y = \frac{48}{5}\ (or\ 9.6)$

$\therefore$ The coordinates of D are $\left(\frac{31}{5}; \frac{48}{5}\right)$ or $(6.2;\ 9.6)$

PAST EXAMINATION QUESTIONS

ZIMSEC

Questions 1

UCLES JUNE 2007 PAPER 1

The vertices of the triangle ABC are $A(-3,1), B(10,-8)$ and $C(1,4)$. Find an equation of the line passing through A and B, giving your answer in the form $px + qy + r = 0$, where p, q and r are integers. **[3]**

Show by calculation that CA and CB are perpendicular. **[2]**

Answer: $13y + 9x + 14 = 0$

The product of gradients $= -1$

Questions 2

UCLES NOVEMBER 1997 PAPER 1

The curve $y = \frac{1}{4}x^2 - 1$ and the line $2y = x + 10$ intersect at the points A and B, and O is the origin. Calculate the coordinates of A and B, and hence show that OA ad OB are peperndicular. **[6]**

Answer: $(-4; 3)$ and $(6; 8)$. The product of gradients $= -1$.

Questions 3

UCLES NOVEMBER 1998 PAPER 1

The straight line L passes through the point $(3, -2)$ and is perpendicular to the line $x + 2y + 1 = 0$. Find the equation of L, giving your answer in the form $y = mx + c$. **[3]**

Answer: $y = 2x - 8$

Questions 4

UCLES JUNE 2000 PAPER 1

Find the equation of the straight line passing through the origin and perpendicular to the line $x + 2y = 3$. **[3]**

Answer: $y = 2x$

Questions 5

UCLES NOVEMBER 2001 PAPER 1

The points P, Q have coordinates $(2, -1), (4, 5)$ respectively. The line L passes through the mid-point of PQ and is parallel to the line with equation $2x + y + 1 = 0$. Find the equation of L, giving your answer in the form $y = mx + c$. **[3]**

Answer: $y = -2x + 8$

Questions 6

ZIMSEC JUNE 2004 PAPER 1

The line l_1 has equation $4x + 3y - 6 = 0$. The line l_2 passes through the point $(0,3)$ and is perpendicular to l_1.

(i) Find the equation of l_2 in the form $y = mx + c$. **[2]**

(ii) The line l_1 crosses the x −axis at A and the y −axis at B. Find the coordinates of A and of B. Hence determine the area of triangle AOB, where O is the origin. **[2]**

Answer: $y = \frac{3}{4}x + 3$

$A\left(\frac{3}{2}; 0\right)$ and $B(0,2)$; Area $\frac{3}{2}$ units^2

Questions 7

ZIMSEC JUNE 2006 PAPER 1

A, B and C are points $(-5;3), (7;-5)$ and $(5;5)$ respectively.

(i) Find the equation of the line which passes through A and B, giving your answer in the form $ax + by + c = 0$. **[2]**

(ii) Given that D is the mid-point of AB, show that CD is perpendicular to AB. **[3]**

(iii) Hence or otherwise show that the area of ΔABC is 52 units^2. **[3]**

Answer: $3y + 2x + 1 = 0$

$D(1;-1)$ and the products of gradients $= -1$

Questions 8

ZIMSEC JUNE 2007 PAPER 1

Line l has equation $3y + 2x = 8$ and point A has coordinates $(1;15)$.

Show that the line from A perpendicular to L has equation $2y - 3x = 27$. **[3]**

Hence find

(i) the perpendicular distance of A from l, **[5]**

(ii) the equation of the circle which has centre A and touches l. **[3]**

Questions 9

ZIMSEC JUNE 2009 PAPER 1

The line $y = 2x + 1$ intersects $y^2 - xy = 3$ at A and B. Find the coordinates of the mid-point of AB. **[5]**

Answer: $\left(-\frac{3}{4}; -\frac{1}{2}\right)$

Questions 10

ZIMSEC JUNE 2014 PAPER 1

The equation of a straight line l_1 is $x + 3y - 33 = 0$. The straight line l_2 is parallel to l_1 and passes through $P(3; 0)$.

(i) Find the equation of l_2. **[2]**

(ii) Show that the line joining P to $Q(6;\ 9)$ is perpendicular to l_1. **[3]**

(iii) Given that the point $R(9;\ -2)$ lies on l_2, and that Q lies on l_1, find the equation of circle passing through P, Q and R. **[3]**

Answer: $3y + x = 3$

The products of gradients $= -1$

$x^2 + y^2 - 15x - 7y + 11 = 0$

Questions 11

ZIMSEC JUNE 2016 PAPER 1

Find the set of values of m such that the gradient of the line passing through the points $(m; 4)$ and $(1; 3 - 2m)$ is less than 5. **[3]**

Answer: $m < 1 \cup m > 2$

Questions 12

ZIMSEC JUNE 2018 PAPER 1

The straight line L passes through the point $(4; -1)$ and is perpendicular to a line with equation $2x - 3y = 10$.

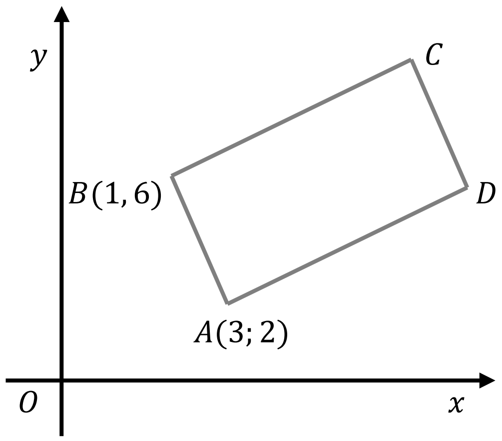
Find the equation of the line L, leaving the answer in the form $ax + by = c$. **[3]**

Answer: $3x + 2y = 10$

CAMBRIDGE QUESTIONS

Questions 1

CAMBRIDGE NOVEMBER 2002 PAPER 1



The diagram shows a rectangle $ABCD$ where A is $(3; 2)$ and B is $(1; 6)$.

(i) Find the equation of BC. **[4]**

Given that the equation of AC is $y = x - 1$, find

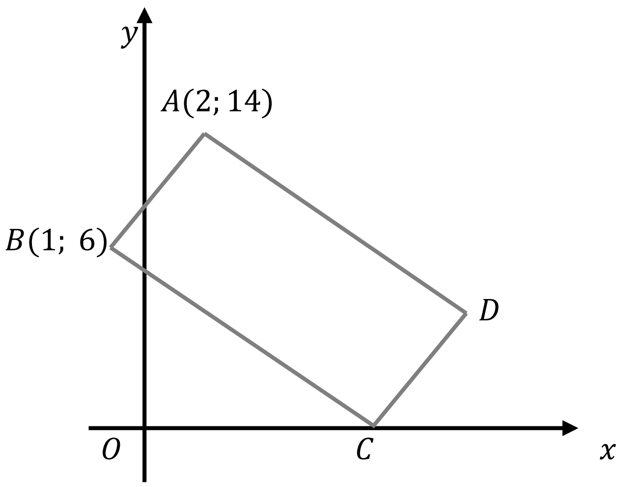
(ii) the coordinates of C, **[2]**

(iii)Find the perimeter of the rectangle $ABCD$. **[3]**

Answer: (i)$2y = x + 11$ (ii)$C(13; 12)$ (iii)35.8 or 35.7 or $16\sqrt{5}$ or $\sqrt{1\ 280}$

Questions 2

CAMBRIDGE JUNE 2007 PAPER 1



The diagram shows a rectangle $ABCD$. The point A is $(2; 14)$, B is $(-2, 8)$ and C lies on the x-axis.

Find

(i) the equation of BC, **[4]**

(ii) the coordinates of C and D. **[3]**

Answer: (i)$3y + 2x = 20$ (ii)C$(10; 0)$ and D$(14; 6)$

Questions 3

CAMBRIDGE JUNE 2017 PAPER 1

The point$A(2; 2)$ lies on the curve $y = x^2 - 2x + 2$.

(i) Find the equation of the tangent to the curve at A. **[3]**

The equation of the curve at A intersects the curve again at B.

(ii) Find the coordinates of B. **[4]**

The tangents at A and B intersect at each other at C

(iii)Find the coordinates of C. **[4]**

Answer: (i)$y = 2x - 2$ (ii) $\left(-\frac{1}{2}; 3\frac{1}{4}\right)$ (iii) $\left(\frac{3}{4}; -\frac{1}{2}\right)$

Questions 4

CAMBRIDGE NOVEMBER 2017 PAPER 1

The point$A(1;1)$ and $B(5;9)$lie on the curve $6y = 5x^2 - 18x + 19$.

(i) Show that the equation of the perpendicular bisector of AB is $2y = 13 - x$. **[4]**

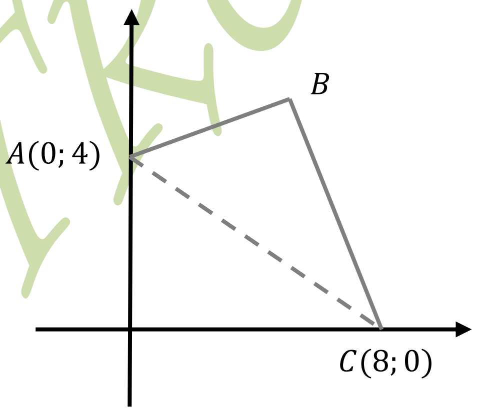
The perpendicular bisector of AB meets the curve at C and D

(ii) Find by calculation, the distance CD, giving your answer in the form $\sqrt{\left(\frac{p}{q}\right)}$ where p and q are integers. **[5]**

Answer: $\sqrt{\left(\frac{125}{4}\right)}$

Questions 5

CAMBRIDGE JUNE 2018 PAPER 1



The diagram shows a kite $OABC$ in which AC is the line of symmetry. The coordinates of A and C are $(0;4)$ and $(8;0)$ respectively and O is the origin.

(i) Find the equation of AC and OB. **[4]**

(ii) Find by calculation the coordinates of B. **[3]**

Answer: (i)$AC: y = -\frac{1}{2}x + 4$, $OB: y = 2x$ (ii)$B(3.2; 6.4)$

Questions 6

CAMBRIDGE NOVEMBER 2018 PAPER 1

The points A and B have coordinates $(3a; -a)$ and $(-a; 2a)$ respectively, where a is a positive constant.

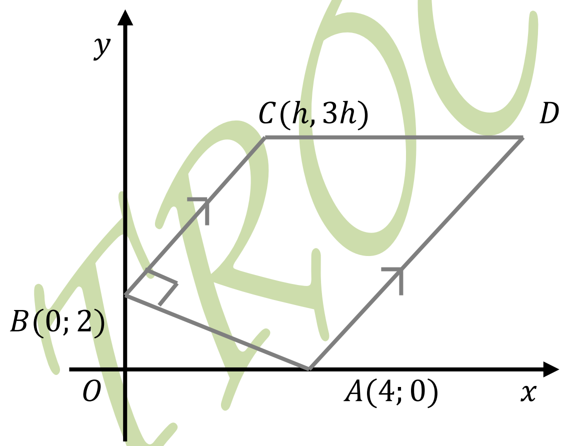
(i) Find the equation of the line through the origin parallel to AB. **[2]**

(ii) The length of the line AB is $3\frac{1}{3}$ units. Find the value of a. **[3]**

Answer: (i)$y = -\frac{3}{4}x$ (ii) $a = \frac{2}{3}$

Questions 7

CAMBRIDGE JUNE 2019 PAPER 11



The diagram shows a trapezium $ABCD$ in which the coordinates of A, B and C are $(4; 0)$, $(0; 2)$ and $(h; 3h)$ respectively. The lines BC and AD are parallel, angle $ABC = 90°$ and CD is parallel to the x-axis.

(i) Find, by calculation, the value of h. **[3]**

(ii) Hence, find the coordinates of D. **[3]**

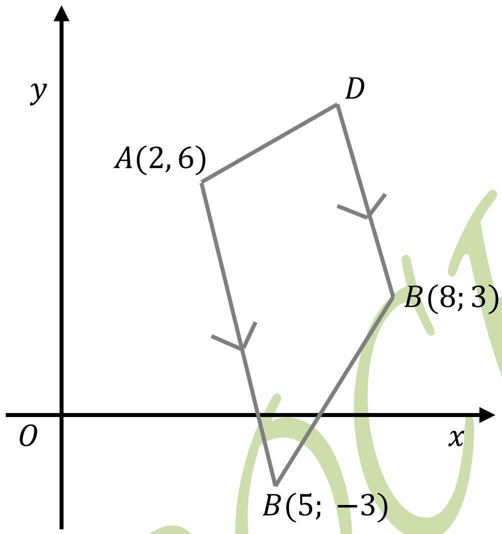
(iii)It is given that AC and BC are equal in lengths. Find an equation relating x and y and show that it can be simplified to $y = 2x - 9$. **[3]**

(iv)Using the results from parts **(ii)** and **(iii)**, and showing all necessary working, find the possible coordinates of C. **[4]**

Answer: (i)$D(5;1)$ (ii)$(x-5)^2 + (y-1)^2 = 20$ (iv)$(3;-3), (7;5)$

<u>Questions 8</u>

<u>CAMBRIDGE JUNE 2019 PAPER 11</u>



The diagram shows a trapezium $ABCD$ in which AB is parallel to DC and angle BAD is 90° . The coordinates of A, B and C are (2,6), (8,3) and (8,3) respectively

(i) Find the equation of AD. **[3]**

(ii) Find, by calculation, the coordinates of D. **[3]**

The point E is such that $ABCE$ is a parallelogram.

(iii)Find the length of BE. **[2]**

Answer: (i)$3y = x + 16$ (ii)$\left(6\frac{1}{2}; 7\frac{1}{2}\right)$ (iii)$BE = 15$

Questions 9

CAMBRIDGE JUNE 2019 PAPER 12

The coordinates of points A and B are $(1,3)$ and $(9,-1)$ respectively and D is the mid-point of . A point C has coordinates $(x;y)$, where x and y are variables.

(i) State the coordinates of D. **[1]**

(ii) It is given that $CD^2 = 20$. Write down an equation relating x and y. **[1]**

(iii)It is given that AC and BC are equal in lengths. Find an equation relating x and y and show that it can be simplified to $y = 2x - 9$. **[3]**

(iv)Using the results from parts (**ii**) and (**iii**), and showing all necessary working, find the possible coordinates of C. **[4]**

Answer: (i)$D(5;1)$ (ii)$(x-5)^2 + (y-1)^2 = 20$ (iv)$(3;-3),(7;5)$

Questions 10

CAMBRIDGE NOVEMBER 2019 PAPER 1

A straight line has gradient m and passes through the point $(0;-2)$. Find the values of m for which the line is a tangent to the curve $y = x^2 - 2x + 7$ and for each value of m find the coordinates of the point where the line touches the curve. **[7]**

Answer: $m = 4$ or -8; $(3;10),(-3,22)$

ASANTE SANA

*******THERE IS A LIGHT AT THE END OF EVERY TUNNEL *******

CONSTRUCTIVE COMMENTS ON THE FORM OF THE PRESENTATION, INCLUDING ANY OMISSIONS OR ERRORS, ARE WELCOME.

ENJOY

Nyasha P. Tarakino (Trockers)

+263772978155/+263717267175

ntarakino@gmail.com