Relations & Functions

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!!!TROCKERS @32!!!

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ACCEPT THIS DOCUMENT AS A GIFT FROM TROCKERS.

I TRULLY THANK GOD FOR TAKING ME THIS FAR (32 years) &
FOR HIS GOODNESS AND BLESSINGS ON ME.

PROVERBS 16vs3

SYLLABUS (6042) REQUIREMENTS

- define a relation
- define domain, co-domain and range
- describe types of relations
- define a function
- distinguish among the following types of functions: one to one, bijective and surjective
- find the inverse of a given function
- illustrate in graphical terms the relation between a one to one function and its inverse
- find composite functions
- define the function as a mapping
- sketch simple graphs of functions for a given domain

RELATIONS

Definition of Terms

Relation: A relation is a rule that relates values from a set of values (domain) to a second set of values (range). It is a set of ordered pair $(X; Y)$. X - is called input, independent variable, domain or abscissa. Y – is called the output, dependent variable, range or ordinate. Simply it is a mapping, or pairing, of input values with output values

Mapping: An illustration showing how each element of the domain is paired with another element of the range in a relation

Domain: The set of all the independent variables or all possible/allowable input values of a relation which produces a real solution. It is represented by X

Co-domain: The set of all dependent variables or all possible or achieved output values of a relation. It is represented by Y.

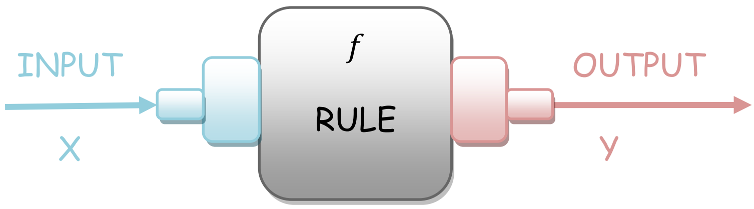
Range: The set of all dependent variables or all actual output values of a relation. It is represented by Y.

Illustration

$f(x) = x^2$

Mapping

$\boldsymbol{x}: (Domain)$ $\sqrt{\boldsymbol{x}}: (Relation)$ $\boldsymbol{f(x)}: (Range)$



Example

Suppose $f(x) = 2x$ then:

x	$f(x)$
0	0
1	2
2	4

Domain : $-\infty \leq x \leq +\infty$

Co − domain : $-\infty \leq y \leq +\infty$

Range : *all even numbers*

MAIN TIPS

Domain

1. For fractions the Denominator should be NON-ZERO
2. For square roots, $\sqrt{f(x)}$, $f(x)$should be NON-NEGATIVE i.e.$f(x) \geq 0$
3. For natural logarithms, $In[f(x)]$, $f(x)$ should be strictly greater than Zero $[f(x) > 0]$
4. For quadratic functions of the form $ax^2 + bx + c$, x takes all values ($x \in \Re$)

Range

1. For square roots, $\sqrt{f(x)}$, $f(x)$should be NON-NEGATIVE i.e.$f(x) \geq 0$
2. $f(x) = \frac{1}{g(x)} \neq 0$
3. For quadratic functions of the form $ax^2 + bx + c$, y has a maximum/minimum thus we need to consider the axis of symmetry i.e $-\frac{b}{2a}$. The coordinates at the axis of symmetry are $\left[-\frac{b}{2a}; f\left(-\frac{b}{2a}\right)\right]$ .Now

   (a) when a is positive we have a minimum value and the range is $y \geq f\left(-\frac{b}{2a}\right)$

   (b) when a is negative we have a maximum value and the range is $y \leq f\left(-\frac{b}{2a}\right)$

Solved Problems

Compute domains and ranges of the following functions:

(a) $y = 2x + 1$

(b) $y = \sqrt{5 - 4x}$

(c) $y = \frac{1}{\sqrt{x+3}}$.

(d) $y = In(3x - 1)$

(e) $y = In(x^2)$

(f) $y = x^2 + 2x - 5$

(g) $y = \frac{1}{3x+1}$.

(h) $y = (2x - 1)^{-1} + 5x$

(i) $y = \sqrt{4 - x^2}$

Suggested Solution

(a) $y = 2x + 1$

Domain

$x \in \Re$

Range

$y \in \Re$

(b) $y = \sqrt{5 - 4x}$

Domain

$5 - 4x \geq 0 \quad \Rightarrow \quad x \leq \frac{5}{4}$

$\therefore x \in \Re: x \leq \frac{5}{4}$ or $\left(-\infty, \frac{5}{4}\right]$

Range

$y \in \Re: y \geq 0$ or $[0, \infty)$

(c) $y = \frac{1}{\sqrt{x+3}}$.

Domain

(i) $\sqrt{x+3} \neq 0$ so $x + 3 \neq 0$ $x \neq -3$

(ii) $x + 3 \geq 0$ $x \geq -3$

combine to obtain $x > -3$

$\therefore x \in \Re: x > -3$ or $(-3, \infty)$

Range

(i) $x + 3 \geq 0 \Rightarrow x \geq -3$ and

(ii) $y = \frac{1}{\sqrt{x+3}} \neq 0$

combine to get $y > 0$

$y \in \Re: y > 0$ or $(0, \infty)$

(d) $y = In(3x - 1)$

Domain

$3x - 1 > 0 \Rightarrow x > \frac{1}{3}$

$\therefore x \in \Re: x > \frac{1}{3}$ or $\left(\frac{1}{3}, \infty\right)$

Range

$y \in \Re$ or $(-\infty, \infty)$

(e) $y = In(x^2)$

Domain

(i) $x^2 > 0 \Rightarrow x > 0$

(ii) $x^2 < 0 \Rightarrow x < 0$

combine to get $x \neq 0$

$\therefore x \in \Re: x \neq 0$ or $(-\infty, 0) \cup (0, \infty): x \neq 0$

Range

$y \in \Re$ or $(-\infty, \infty)$

(f) $y = 3x^2 + 6x - 3$

First complete the square:

$$3x^2 + 6x - 3 \equiv 3(x^2 + 2x - 1) \equiv 3[x^2 + 2x + (+1)^2 - (+1)^2 - 1]$$

$$3[(x+1)^2 - 1 - 1]$$

$$3[(x+1)^2 - 2]$$

$$3(x+1)^2 - 6$$

Find axis of symmetry and state its nature:

$$x = -\frac{b}{2a} = -\frac{6}{2(3)} = -1 \text{ and it is a minimum turning point}$$

<u>Domain</u>

$\therefore x \in \Re$ or $(-\infty, \infty)$

<u>Range</u>

$y \in \Re$ or $(-1, \infty)$

(g) $y = \frac{1}{3x+1}$.

<u>Domain</u>

$3x + 1 \neq 0$ so $x \neq -\frac{1}{3}$

$\therefore x \in \Re : x \neq -\frac{1}{3}$ or $\left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, \infty\right) : x \neq -\frac{1}{3}$

<u>Range</u>

$y \in \Re : x \neq 0$ or $(-\infty, 0) \cup (0, \infty) : y \neq 0$

(h) $y = (2x - 1)^{-1} + 5x$

Note:

$(2x-1)^{-1} - 5x \Rightarrow \frac{1}{2x-1} + 5x$

<u>Domain</u>

$2x - 1 \neq 0$ so $x \neq \frac{1}{2}$

$\therefore x \in \Re : x \neq \frac{1}{2}$ or $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right) : x \neq \frac{1}{2}$

Range

$y \in \Re: x \neq 0$ or $(-\infty, 0) \cup (0, \infty): y \neq 0$

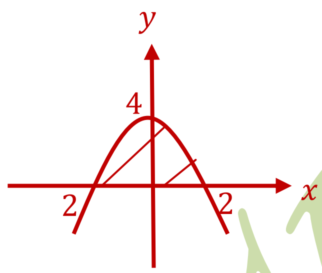
(i) $y = \sqrt{4 - x^2}$

Domain

$4 - x^2 \geq 0$

$(2 - x)(2 + x) \geq 0$

Critical values: $x = \pm 2$



Since the inequality sign is ≥ 0 which means that the required solutions are those above the $x - axis$.

$$x \in \Re: -2 \leq x \leq 2$$

$\therefore$ The domain of $gf(x) = x \in \Re: -2 \leq x \leq 2$

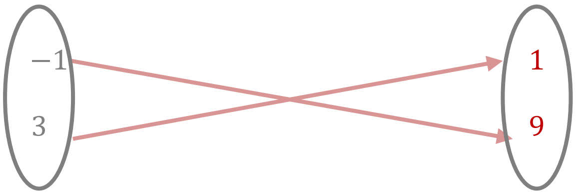
Range

$y \in \Re: y \leq 4$ or $[4, \infty)$

TYPES OF RELATIONS

One to One

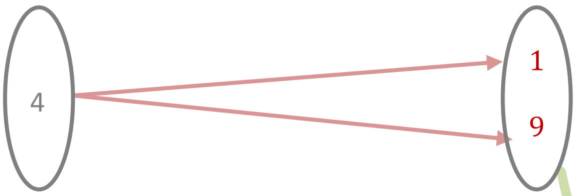
Mapping



x: (Domain) x^2: (Relation) $f(x)$: (Range)

One to Many

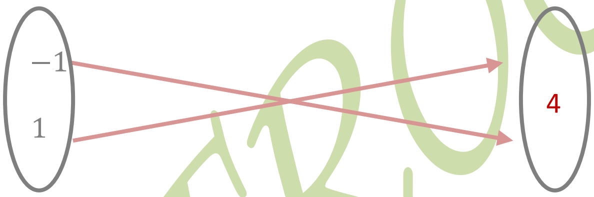
Mapping



x: (Domain) $\sqrt{x}$: (Relation) $f(x)$: (Range)

Many to One

Mapping



x: (Domain) x^2: (Relation) $f(x)$: (Range)

Many to Many

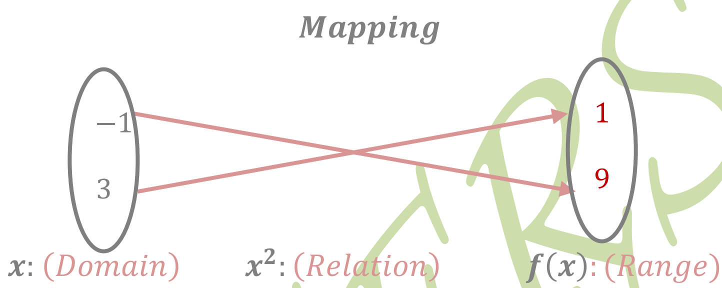
Mapping

x: (Domain) (Relation) $f(x)$: (Range)

FUNCTIONS

Definition

Function: A relation where each element of set A (the domain) is paired with exactly one element of set B (the range). In other words, a function f is a special type of relation in which each input has exactly one output.



FUNCTION NOTATION

- Function notation is a way to name a function that is defined by an equation.
- For an equation in x and y, the notation $f: X \rightarrow Y$ means that f *is* a function from X to Y.
- X is called the domain of f and Y is called the co-domain (range) of f.
- Given an element $x \in X$, there is a unique element y in Y that is related to x. The unique element y to which f relates x is denoted by $f(x)$ and is called f of x, or the value of f at x, or the image of x under f.
- The set of all values of $f(x)$ taken together is called the range of f or image of X under f.
- The symbol $f(x)$ replaces y and is read as "the value of the function at x" or simply "f of x".
- Remember, that the x value is the independent variable and the y value is dependent on what the value of x is.
- So, y is a function of $y = f(x)$ or in other words y and the function of x are interchangeable.
- Thus in general:
  - x is the input
  - f is the function operator
  - y or $f(x)$ is the output

FUNCTION REPRESENTATION

A function can be represented by an equation that describes the mathematical relationship that exists between the independent (x) and dependent (y) variables e.g. $y = x^2 - 1$ or other several different ways i.e. ordered pairs, table of values, mapping diagrams and graphs.

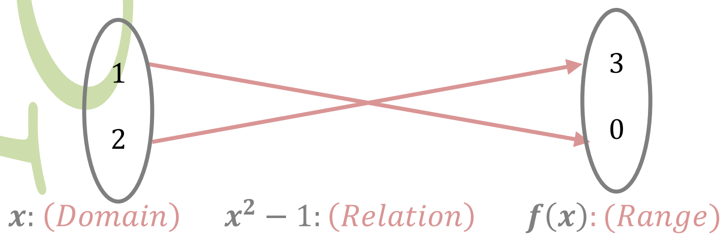
- **Ordered Pairs:** given a relation, it is a function if each input is paired with exactly 1 output (check to see if x repeats). For example $y = x^2 - 1$

  $(-1,0)$ $(0,-1)$ $(1,0)$ $(2,3)$

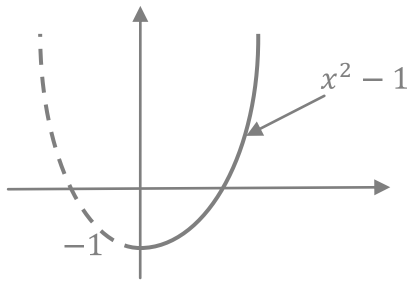
- **Table of Values:** given a table of values of a relation, it is a function if each input is paired with exactly one output (check to see if x repeats). For example $y = x^2 - 1$

x	-1	0	1	2
y	0	-1	0	3

- **Mapping Diagrams:** given a mapping diagram of a relation, we can tell if it is a function if each input is paired with exactly one output. For example $y = x^2 - 1$



- **Graph:** given a graph, it is a function if each input is paired with exactly one output (check to see if x repeats). For example $y = x^2 - 1$



FUNCTION EVALUATION

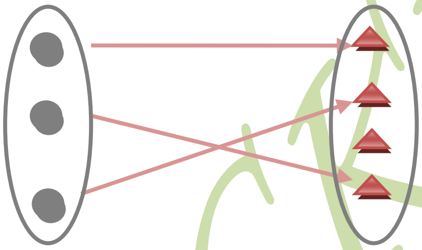
Sometimes a function has an output formula given by $f(x)$ then to evaluate the output for $f(x)$, given an input: We just plug in the input, wherever we see $f(x)$.

Types of functions

There are three types of functions namely Injective, Surjective and Bijective.

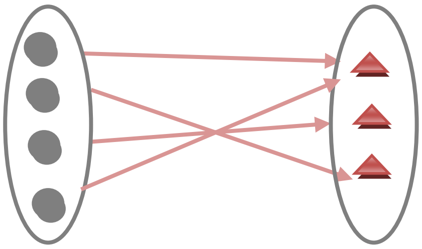
One to one (Injective)

$f(x)$ is said to be Injective (one-one) if no two inputs have the same output i.e. For every input there is a unique output. All the elements in the domain have to be used, but all elements in the co-domain need not be used. Its inverse is also function



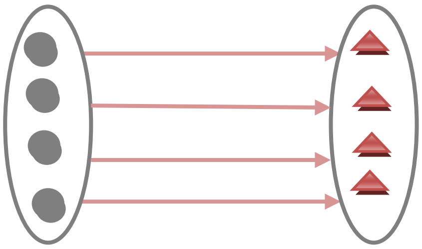
Surjective (Onto)

$f(x)$ is said to be Surjective (onto) if every element in the co-domain (Set B) has some element from the domain (Set A) mapped to it. Each element of B has its pre-image in A. Every element in the co-domain is the image of at least one element in the domain. If it is 'onto' then everything on the co-domain must have something mapped onto it. Its inverse is not always a function.



Bijective

$f(x)$ is said to be Bijective if it is both Injective and Surjective.



Vertical and Horizontal Line Test

Vertical Line Test

- Given the graph, we can use the "vertical line test" to determine if a relation is function.
- If every vertical line intersects the graph at no more than one point, then the graph represents the function
- If every vertical line intersects the graph twice, then the graph represents a relation and is not a function

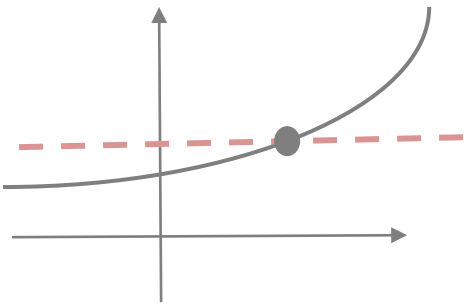
Horizontal Line Test

- Given the graph, we can use the "horizontal line test" to determine if a relation is a one-one function (Injective)/ Surjective function.
- A relation is a one-one relation if and only if no horizontal line intersects the graph of the relation at more than one point.
- The graph is a one-one function if a horizontal line is drawn and intersects it exactly once (each time).
- If the horizontal line intersects the graph at more than one point, the graph is NOT a one-one function.

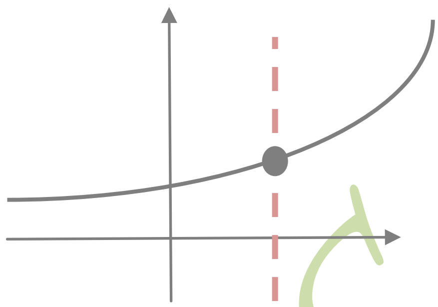
NB: If a graph passes the horizontal test it is a function and if it passes the horizontal test it is injective

Illustration

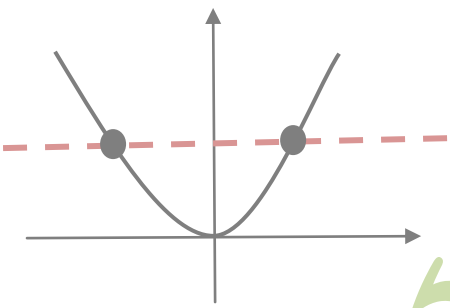
The graph represents a one-one function



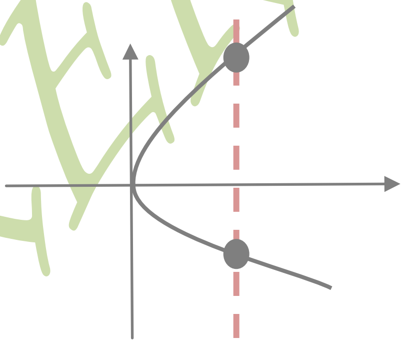
The graph represents a function



The graph represents a function (not one-one)



The graph is a relation not a function and it is a one-one relation

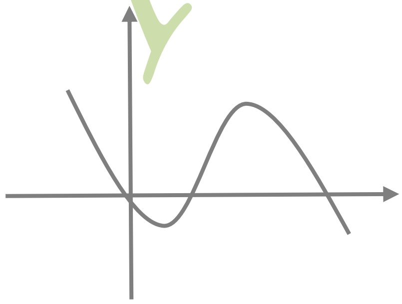


Solved Problems

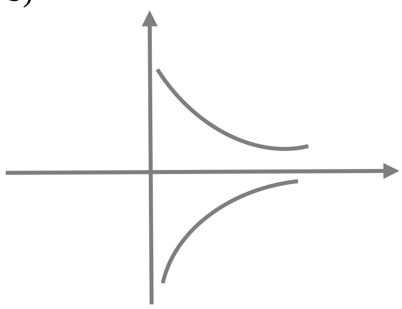
Question 1

Determine whether the following graphs represent a function or a relation and state if it is Surjective or Injective.

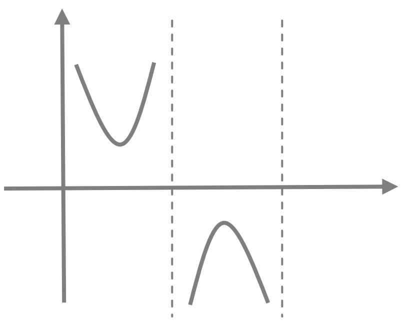
a)



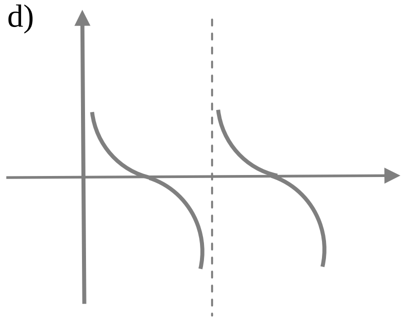
b)



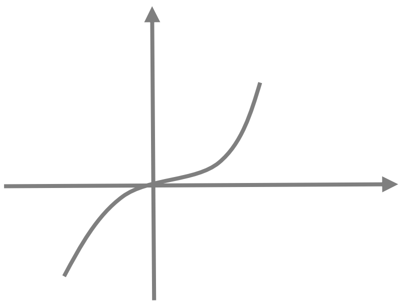
c)



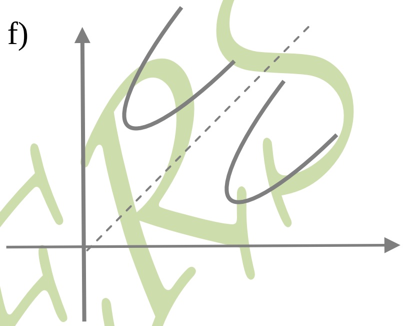
d)



e)



f)



Suggested Solution

(a) Surjective function

(b) Injective relation

(c) Surjective function

(d) Surjective function

(e) Injective function

(f) Surjective relation

Question 2

Sketch, on separate diagrams, graphs represented by the following functions. Hence state the range in each case.

(a) $y = 3x + 1, \quad -1 \leq x \leq 3$

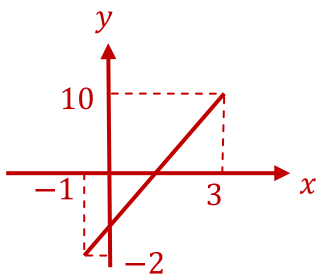
(b) $y = In(1 - 2x), \quad -3 \leq x \leq 0$

(c) $y = 3x^2 + 2x - 1, \quad 0 \leq x \leq 2$

(d) $y = \sqrt{5x + 2}\ for\ 0 \leq x \leq 5$

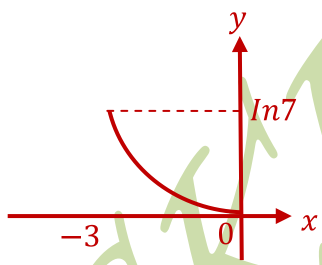
Suggested Solution

(a) $y = 3x + 1, \quad -1 \leq x \leq 3$



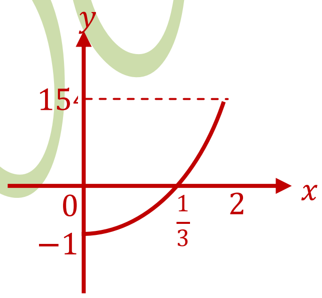
Range: $y \in \mathbb{R}: -12 \leq y \leq 10$

(b) $y = In(1-2x), \quad -3 \leq x \leq 0$



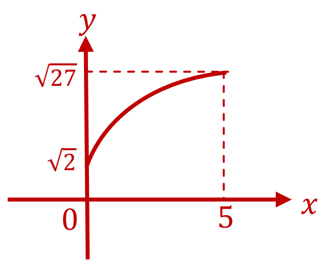
Range: $y \in \mathbb{R}: 0 \leq y \leq In7$

(c) $y = 3x^2 + 2x - 1, \quad 0 \leq x \leq 2$



Range: $y \in \mathbb{R}: -1 \leq y \leq 15$

(d) $y = \sqrt{5x+2}\ for\ 0 \leq x \leq 5$



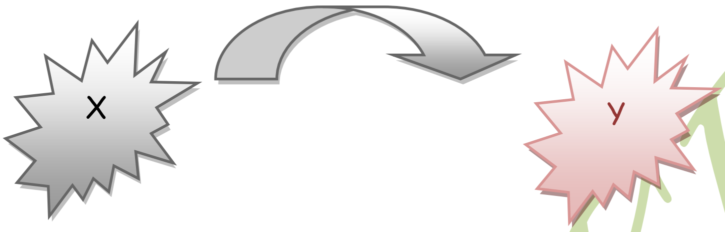
Range: $y \in \mathbb{R}: \sqrt{2} \leq y \leq \sqrt{27}$

THE INVERSE OF A FUNCTION

- The inverse of a function undo or reverse the result of that function. It is the opposite of the original function.
- The inverse of $f(x)$ is denoted by $f^{-1}(x)$
- Bijective functions have inverses and the inverse on an Injective function is also a function
- Geometrically it is found by reflecting the function (or relation) about the line $y = x$.

NB: $f^{-1}(x) \neq \frac{1}{f(x)}$

<u>Function</u>



<u>Inverse</u>

- The inverse of a function $f(x)$ is denoted by $f^{-1}(x)$.
- Two methods are going to be employed i.e.

(a) Interchanging x and y and

<u>Steps</u>

- replace $f(x)$ with y
- interchanging x and y
- solve for y
- replace y with $f^{-1}(x)$

(b) The flow chart diagram.

<u>Steps</u>

- start with x

- write down the sequence of all transformations for which x undergoes to produce $f(x)$
- write x again
- write these sequential transformations but in reverse order to produce $f^{-1}(x)$

Solved Problems

Question

Find the inverse of the following functions and clearly state the domains:

(a) $y = 3x + 1$

(b) $y = \sqrt{2 - 5x}$

(c) $y = In(8x + 7)$

(d) $y = x^2 - 6x + 10$

(e) $y = \frac{1}{3x+2}$.

(f) $y = e^{x-4}$

Suggested Solution

(a) $y = 3x + 1$

Method 1

Interchange x and y and make y the subject:

Let $x = 3y + 1$

$\Rightarrow 3y = x - 1$

$$y = \frac{x-1}{3}$$

$$\therefore f^{-1}(x) = \frac{1}{3}(x - 1): x \in \Re$$

Method 2

Write down all the stages for which x undergoes to produce $f(x)$. Then for the inverse start with x and write the stages above in reverse.

$y = 3x + 1$

Now:

Generating the Function:

$$x \xrightarrow{(\times 3)} 3x \xrightarrow{(+1)} f(x) = 3x + 1$$

START STOP

Generating the Inverse:

$$f^{-1}(x) = \frac{x-1}{3} \xleftarrow{(\div 3)} x - 1 \xleftarrow{(-1)} x$$

STOP START

$$\therefore f^{-1}(x) = \frac{1}{3}(x - 1) : x \in \Re$$

(b) $y = \sqrt{2 - 5x}$

Method 1

Interchange x and y and make y the subject:

Let $x = \sqrt{2 - 5y}$

$\Rightarrow 2 - 5y = x^2$

$$y = \frac{2 - x^2}{5}$$

$$\therefore f^{-1}(x) = \frac{1}{5}(2 - x^2) : x \in \Re$$

Method 2

Write down all the stages for which x undergoes to produce $f(x)$. Then for the inverse start with x and write the stages above in reverse.

$y = \sqrt{2 - 5x}$

Now:

Generating the Function:

$$x \xrightarrow{(\times -5)} -5x \xrightarrow{(+2)} 2 - 5x \xrightarrow{\textit{Square root}} f(x) = \sqrt{2 - 5x}$$

START STOP

Generating the Inverse:

$$f^{-1}(x) = \frac{1}{5}(2-x^2) \xleftarrow{(\div -5)} x^2 - 2 \xleftarrow{(-2)} x^2 \xleftarrow{Square} x$$

STOP START

$$\therefore f^{-1}(x) = \frac{1}{5}(2-x^2) : x \in \Re$$

(c) $y = In(8x+7)$

<u>Method 1</u>

Interchange x and y and make y the subject:

Let $x = In(8y+7)$

$\Rightarrow 8y + 7 = e^x$

$8y = e^x - 7$

$$y = \frac{e^x - 7}{8}$$

$$\therefore f^{-1}(x) = \frac{1}{8}(e^x - 7) : x \in \Re$$

<u>Method 2</u>

Write down all the stages for which x undergoes to produce $f(x)$. Then for the inverse start with x and write the stages above in reverse.

$y = In(8x+7)$

Now:

Generating the Function:

$$x \xrightarrow{(\times 8)} 8x \xrightarrow{(+7)} 8x + 7 \xrightarrow{Natural\ Log} f(x) = In(8x+7)$$

START STOP

Generating the Inverse:

$$f^{-1}(x) = \frac{1}{8}(e^x - 7) \xleftarrow{(\div 8)} e^x - 7 \xleftarrow{(-7)} e^x \xleftarrow{Exponent} x$$

STOP START

$$\therefore f^{-1}(x) = \frac{1}{8}(e^x - 7): x \in \Re$$

(d) $y = x^2 - 6x + 10$

First complete the square:

$$x^2 - 6x + 10 \equiv x^2 - 6x + (-3)^2 - (-3)^2 + 10$$
$$= (x-3)^2 - 9 + 10$$
$$= (x-3)^2 + 1$$

Method 1

Interchange x and y and make y the subject:

Let $x = (y-3)^2 + 1$

$\Rightarrow (y-3)^2 = x - 1$

$y - 3 = \sqrt{x-1}$

$y = 3 + \sqrt{x-1}$

$\therefore f^{-1}(x) = 3 + \sqrt{x-1}: x \in \Re; x \geq 1$

Method 2

Write down all the stages for which x undergoes to produce $f(x)$. Then for the inverse start with x and write the stages above in reverse.

$y = (x-3)^2 + 1$

Now:

Generating the Function:

$$x \xrightarrow{(-3)} x - 3 \xrightarrow{Square} (x-3)^2 \xrightarrow{+1} f(x) = (x-3)^2 + 1$$

START STOP

Generating the Inverse:

$$f^{-1}(x) = 3 + \sqrt{x-1} \xleftarrow{(+3)} \sqrt{x-1} \xleftarrow{Square\ root} x - 1 \xleftarrow{-1} x$$

STOP START

$\therefore f^{-1}(x) = 3 + \sqrt{x-1}: x \in \Re; x \geq 1$

(e) $y = \frac{1}{3x+2}$.

Method 1

Interchange x and y and make y the subject:

Let $x = \frac{1}{3y+2}$

$\Rightarrow x(3y+2) = 1$

$3xy + 2x = 1$

$3xy = 1 - 2x$

$$y = \frac{1-2x}{3x}$$

$$\therefore f^{-1}(x) = \frac{1-2x}{3x} : x \in \Re; x \neq 0$$

Method 2

Write down all the stages for which x undergoes to produce $f(x)$. Then for the inverse start with x and write the stages above in reverse.

$$y = \frac{1}{3x+2}$$

Now:

Generating the Function:

$$x \xrightarrow{(\times 3)} 3x \xrightarrow{(+2)} 3x+2 \xrightarrow{\textit{Reciprocal}} f(x) = \frac{1}{3x+2}$$

START STOP

Generating the Inverse:

$$f^{-1}(x) = \frac{1-2x}{3x} \xleftarrow{(\div 3)} \frac{1-2x}{x} \xleftarrow{(-2)} y = \frac{1}{x} \xleftarrow{\textit{Reciprocal}} x$$

STOP START

$$\therefore f^{-1}(x) = \frac{1-2x}{3x} : x \in \Re; x \neq 0$$

(f) $y = e^{x-4}$

Method 1

Interchange x and y and make y the subject:

Let $x = e^{y-4}$

$\Rightarrow y - 4 = In(x)$

$y = In(x) + 4$

$\therefore f^{-1}(x) = In(x) + 4 : x \in \Re ; x > 0$

Method 2

Write down all the stages for which x undergoes to produce $f(x)$. Then for the inverse start with x and write the stages above in reverse.

$y = e^{x-4}$

Now:

Generating the Function:

$$x \xrightarrow{(-4)} x - 4 \xrightarrow{Exponential} f(x) = e^{x-4}$$

START STOP

Generating the Inverse:

$$f^{-1}(x) = In(x) + 4 \xleftarrow{(+4)} In(x) \xleftarrow{Natural\ Logarithm} x$$

STOP START

$\therefore f^{-1}(x) = In(x) + 4 : x \in \Re ; x > 0$

The relationship between a one to one function and its inverse

$f(x) = x + 1$

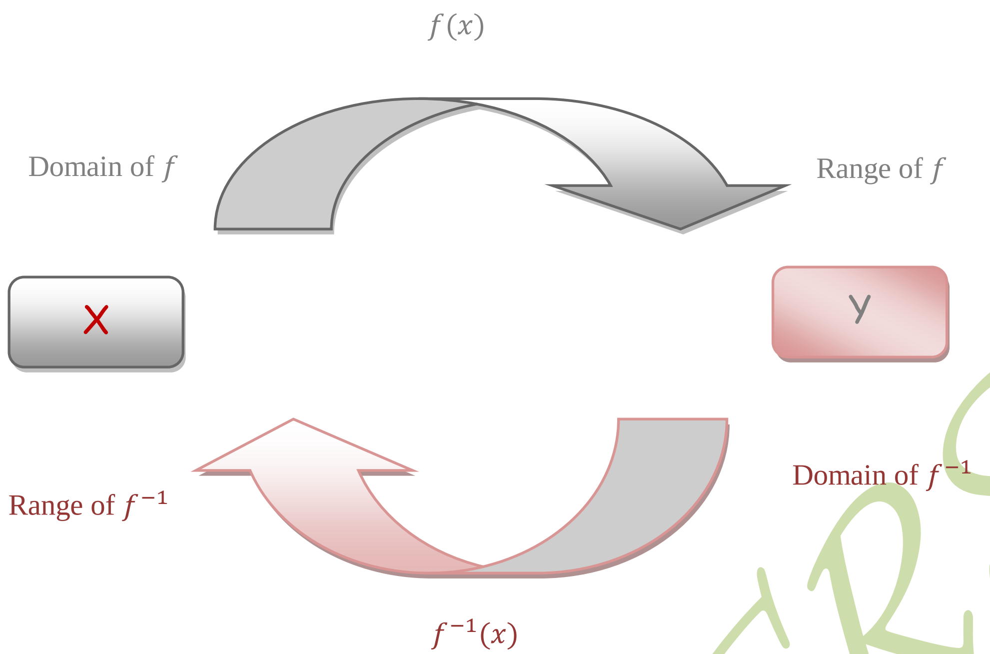
$$\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \begin{array}{c} \longrightarrow \\ \longrightarrow \\ \longrightarrow \end{array} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Domain *Range*

$f^{-1}(x) = x - 1$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{array}{c} \longrightarrow \\ \longrightarrow \\ \longrightarrow \end{array} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

Domain *Range*



From the above illustration it is clear that:

(a) The Domain of the function is the Range of the inverse and

(b) The Domain of the inverse is the Range of the function.

The relationship between the graph of a one to one function and its inverse

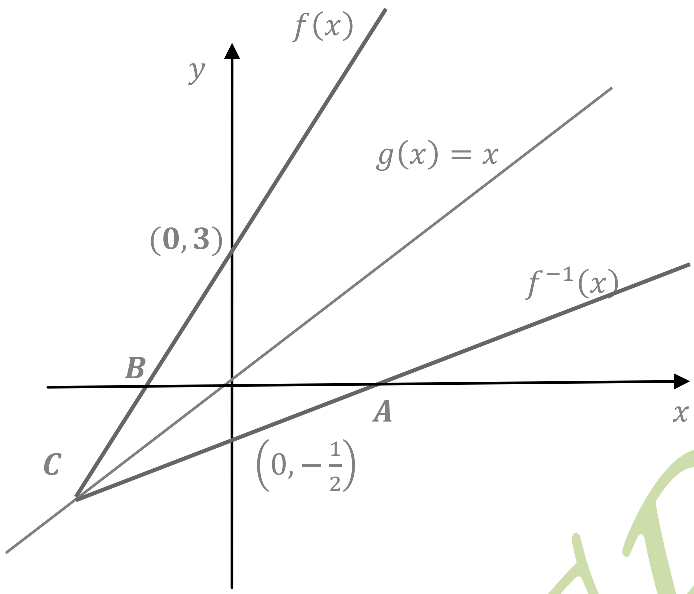
The graph of the function and the graph of its inverse function reflect on the line $y = x$ i.e. the graph of $y = f(x)$ is symmetric to the graph of $y = f^{-1}(x)$ along the line $y = x$.

Solved Problems

Question 1

ZIMSEC NOVEMEBR 2010 PAPER 1

The diagram below shows the linear graphs of $f(x), g(x)$ and $f^{-1}(x)$ which intersect at $\boldsymbol{C}$. The graph of $f^{-1}(x)$ intersects the $x - axis$ at point $\boldsymbol{A}$ and $f(x)$ intersects the $x - axis$ at $\boldsymbol{B}$.



State the name given to the function $g(x)$ in relation to the functions $f(x)$ and $f^{-1}(x)$. **[1]**

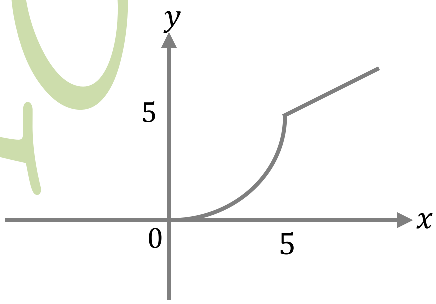
Suggested Solution

Mirror line or line of reflection

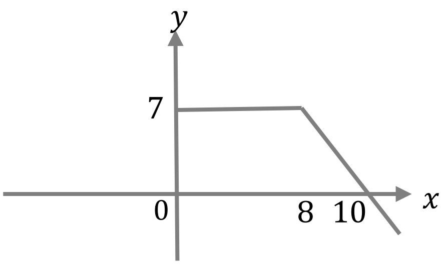
Question 2

The diagrams below show the graphs of $f(x)$. Draw, on separate diagrams, graphs of $f^{-1}(x)$ and state clearly the relationship between the graphs of $f(x)$ and $f^{-1}(x)$.

(a)

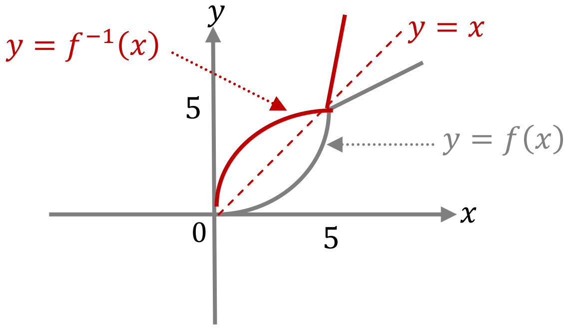


(b)



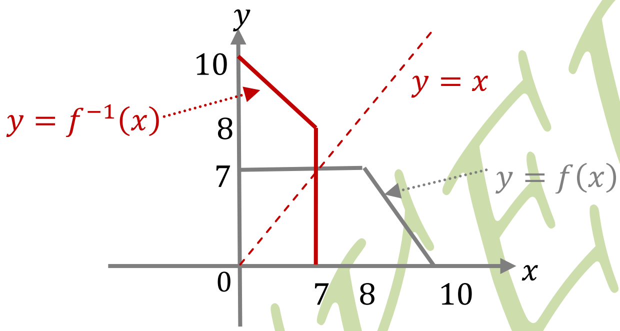
Suggested Solution

(a)



The graphs of the functions $f(x)$ and $f^{-1}(x)$ are reflecting on the line $y = x$

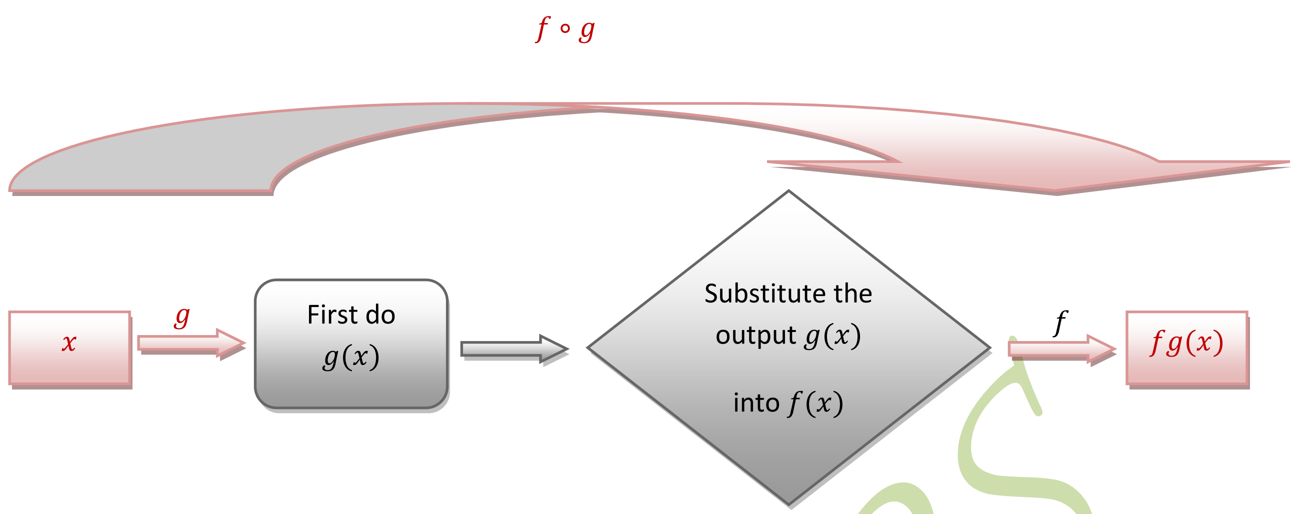
(b)



The graphs of the functions $f(x)$ and $f^{-1}(x)$ are reflecting on the line $y = x$

COMPOSITE FUNCTIONS

- It is a function followed by another function or simply two functions carried out in sequence i.e. the result of one function is applied to a second function
- The composition of function $f(x)$ and $g(x)$ is denoted by $fg(x)$ or $gf(x)$ or $g \circ f$ or $f \circ g$
- $fg(x)$ means do $g(x)$ first, then we put the result into $f(x)$
- For composite functions we always take a right left direction i.e. we start with the function on the right side



NB

(i) $g \circ f \neq f \circ g$

(ii) $g^2 = g \circ g = gg(x)$

(iii) If $fg(x) = x$ then $f(x)$ is the inverse of $g(x)$

Solved Problems

QUESTION 1

Given that $f(x) = x^2 - 1$, $g(x) = e^{2x}$ and $h(x) = In(x + 2)$, find:

(a) $fg(x)$

(b) $gf(x)$

(c) $fhg(x)$

(d) $gh(x)$

(e) $hf(x)$

Suggested Solution

$f(x) = x^2 - 1$, $g(x) = e^{2x}$ and $h(x) = In(x + 2)$.

(a) $fg(x) = [g(x)]^2 - 1$

$= (e^{2x})^2 - 1$

$= e^{4x} - 1 : x \in \Re$

(b) $gf(x) = e^{2[f(x)]}$

$= e^{2x^2 - 2}$

$= e^{2x^2-2}: x \in \Re$

(c) $fhg(x) = f[In(\{g(x)\} + 2)]$

$= f[In(e^{2x} + 2)]$

$= ([In(e^{2x} + 2)])^2 - 1$

$= [In(e^{2x} + 2)]^2 - 1: x \in \Re$

(d) $gh(x) = e^{2[h(x)]}$

$= e^{2[In(x+2)]}$

$= e^{In(x+2)^2}$

$= (x + 2)^2: x \in \Re$

(e) $hf(x) = In[f(x) + 2]$

$= In(x^2 - 1 + 2)$

$= In(x^2 + 1): x \in \Re$

QUESTION 2

It is given that $f(x) = x + 3, x \in \Re$ and $g(x) = In(x - 1), x \in \Re: x > 1$.

(i) Find $gf(x)$ and clearly state its domain.

(ii) Draw the graph of $gf(x)$ and show that $gf(x)$ is a one-one function.

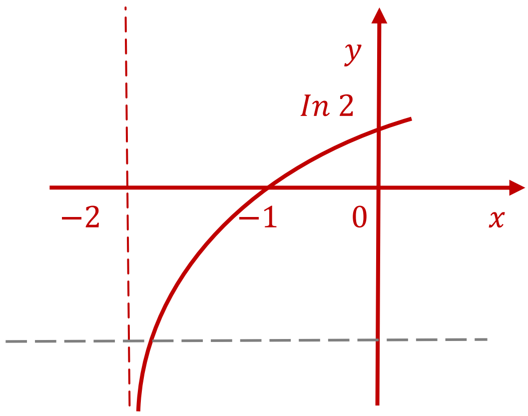
Suggested Solution

(i) $gf(x) = g[f(x)] = In[f(x) - 1]$

$= In(x + 3 - 1)$

$= In(x + 2), x \in \Re: x > -2$

(ii)



Since the horizontal line (In grey) cuts the graph once then $gf(x)$ is a one-one function.

QUESTION 3

Given that $g(x) = 2 - (x+5)^2$ and $f(x) = x^2$

State a sequence of transformations for which the graph of $f(x)$ undergoes to produce the graph of $g(x)$.

Hence sketch the graphs of $f(x)$ and $g(x)$ on the same axis.

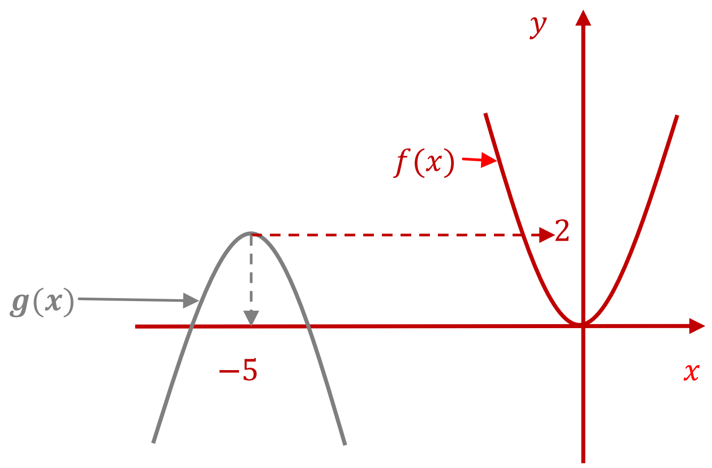
Suggested Solution

$g(x) = 2 - (x+5)^2$ and $f(x) = x^2$

a) Translation along the $x - axis$, moving 5 units leftwards or Translation along the $x - axis$ with translation vector $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$.

b) Reflection on the line $y = 0$ or $x - axis$.

c) Translation along the $y - axis$, moving 2 units upwards or Translation along the $y - axis$ with translation vector $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.



QUESTION 4

Functions f and g are defined by:

$$f: x \to 2x - 5, x \in \mathbb{R}$$

$$g: x \to \frac{4}{2-x}, x \in \mathbb{R}, x \neq 2.$$

(i) Find the value of x for which $fg(x) = 7$.

(ii) Express each of $f^{-1}(x)$ and $g^{-1}(x)$ in terms of x.

(iii) Show that the equation $f^{-1}(x) = g^{-1}(x)$ has no real roots.

(iv) Sketch, on a single diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making a clear relationship between these two graphs.

Suggested Solution

(i) $fg(x) = 2\left(\frac{4}{2-x}\right) - 5 = 7$

$$\Rightarrow \frac{48}{2-x} = 12$$

$$\therefore x = \frac{4}{3}.$$

(ii) Use appropriate method to get

$$g^{-1}(x) = \frac{2x-4}{x}; x \in \mathbb{R}: x \neq 0 \text{ and}$$

$$f^{-1}(x) = \frac{x+5}{2};\ x \in \mathbb{R}.$$

(iii) $\frac{x+5}{2} = \frac{2x-4}{x}$

$\Rightarrow x^2 + 5x = 4x - 8$

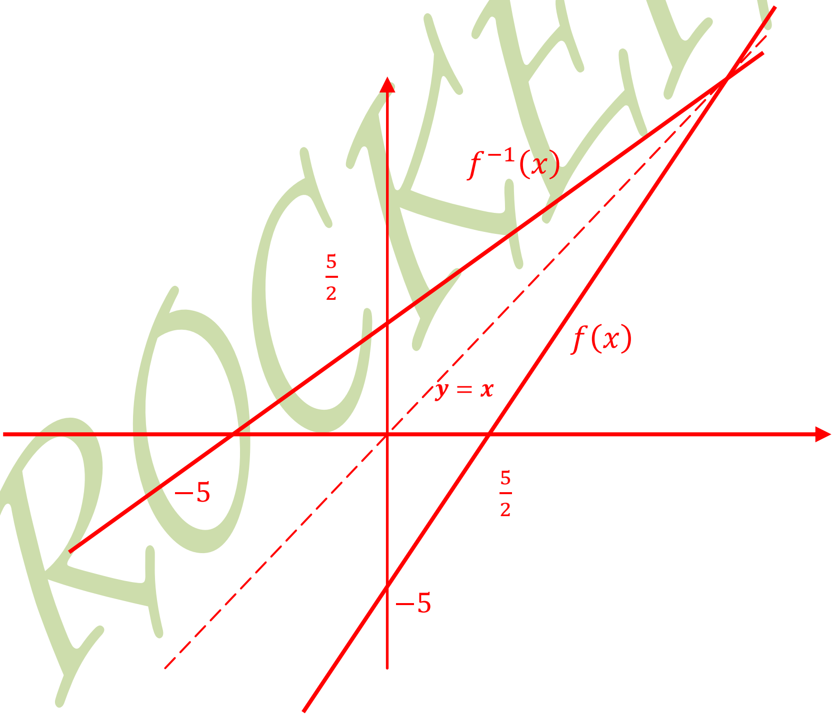
$\Rightarrow x^2 + x + 8 = 0$

Now $b^2 - 4ac = (1)^2 - 4(1)(8) = -31 < 0$

$\therefore$ there are no real roots.

(iv) $y = 2x - 5$: When $x = 0; y = 5$ and when $y = 0; x = \frac{5}{2}$.

The graphs of $y = f(x)$ and $y = f^{-1}(x)$ reflect on the line $y = x$.



QUESTION 5

Given that : $f(x) \to \frac{x+p}{x-3}$, $(x \neq 3)$, where p is a constant.

a) Find the value of p if $f(5) = 1\frac{1}{2}$.

b) Hence

(i) Find $f^{-1}(x)$ in a similar form.

(ii) State the value of x for which $f^{-1}(x)$ is undefined.

Suggested Solution

(a) $f(5) = \frac{5+p}{5-3} = 1\frac{1}{2}$

$\Rightarrow \frac{5+p}{2} = \frac{3}{2}$

$\Rightarrow 5 + p = 3$

$\Rightarrow p = -2.$

(b) If $f(x) = \frac{x-2}{x-3}$ $\Rightarrow y = \frac{x-2}{x-3}$

Now, let $x = \frac{y-2}{y-3}$.

Making y the subject yields $y = \frac{3x-2}{x-1}$

$\therefore f^{-1}(x) \to \frac{3x-2}{x-1}; x \neq 1.$

(c) $x = 1.$

QUESTION 6

The function f is defined by $f: x \mapsto sinx,\ x \in \mathbb{R}: -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

(i) Explain why f has an inverse.

(ii) Sketch the graph of $y = f^{-1}(x)$ and hence state its domain and range.

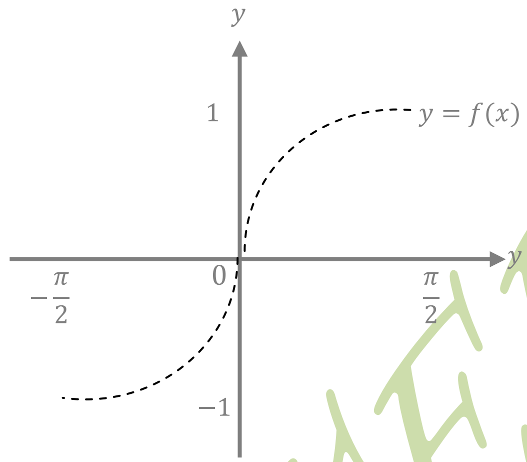
(iii) Given that $g: x \mapsto x + \frac{\pi}{6},\ x \in \mathbb{R}$, solve the equation $2(f \circ g) = 1$ for

$0 \leq x \leq 2\pi$.

Suggested Solution

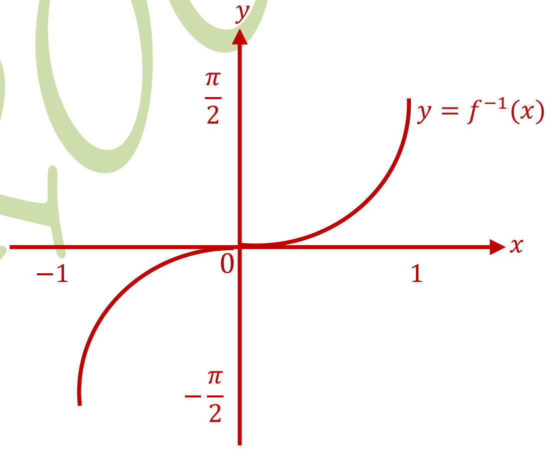
(i) It is an injective function/ one-one function

(ii)



NB: The graph of $y = f^{-1}(x)$ is the reflection of the graph of $y = f(x)$ on

the line $y = x$



Domain: $y \in \mathbb{R}: -1 \leq x \leq 1$

Range: $y \in \mathbb{R}: -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.

NB: Domain of $y = f^{-1}(x)$ is the range of $y = f(x)$ and range of $y = f^{-1}(x)$

is the domain of $y = f(x)$.

(iii) $2(f \circ g) = 1$

$$\Rightarrow fg(x) = \frac{1}{2}$$

$$\Rightarrow sin[g(x)] = \frac{1}{2}$$

$$\Rightarrow sin\left(x + \frac{\pi}{6}\right) = \frac{1}{2}$$

$$\Rightarrow x + \frac{\pi}{6} = sin^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow x + \frac{\pi}{6} = \frac{\pi}{6};\ \pi - \frac{\pi}{6};\ 2\pi + \frac{\pi}{6}$$

$$\Rightarrow x + \frac{\pi}{6} = \frac{\pi}{6};\ \frac{5\pi}{6};\ \frac{13\pi}{6}$$

$$\Rightarrow x = \frac{\pi}{6} - \frac{\pi}{6};\ \frac{5\pi}{6} - \frac{\pi}{6};\ \frac{13\pi}{6} - \frac{\pi}{6}$$

$$\therefore x = 0;\ \frac{2\pi}{3};\ 2\pi$$

SOLVED PAST EXAMINATION QUESTIONS

Question One

ZIMSEC JUNE 2020 PAPER 1

The function $f(x) = 2 - \frac{1}{x}$, $x > 0$

(a) State the graph of $f(x)$ and state the range. **[3]**

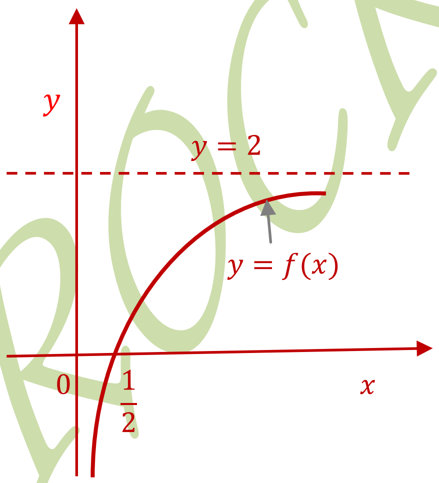
(b) Find $f^{-1}(x)$, the inverse of $f(x)$. **[2]**

(c) Calculate the value of x for which $f(x) = f^{-1}(x)$. **[3]**

Suggested Solution

(a) First sketch the graph of $\frac{1}{x}$ for $x > 0$ and perform the following transformations:

- Reflection along the x-axis $\{-\boldsymbol{f(x)}\}$
- Translation on the y-axis, moving 2 units upwards $\{\boldsymbol{f(x)+a}\}$



Range: $-\infty < f(x) < 2$ or $-\infty < y < 2$ or $f(x) < 2$ or $y < 2$

(b) $f(x) = 2 - \frac{1}{x}$, $x > 0$

Let $x = 2 - \frac{1}{y}$

$\Rightarrow \frac{1}{y} = 2 - x$

$$\Rightarrow \frac{1}{2-x} = y$$

$$\therefore f^{-1}(x) = \frac{1}{2-x}, x \in \mathbb{R}: x < 2$$

(c) $f(x) = f^{-1}(x)$

$$\Rightarrow 2 - \frac{1}{x} = \frac{1}{2-x}$$

$$\Rightarrow x(2-x)2 - x(2-x)\frac{1}{x} = x(2-x)\frac{1}{2-x}$$

$$\Rightarrow 4x - 2x^2 - 2 + x = x$$

$$\Rightarrow 2x^2 - 4x + 2 = 0$$

$$\Rightarrow x^2 - 2x + 1 = 0$$

$$\Rightarrow (x-1)^2 = 0$$

$$\therefore x = 1(twice)$$

Question Two

ZIMSEC JUNE 2020 PAPER 2

Suggested Solution

$f(x) = 4 - x^2, \quad x \in \mathbb{R}$

$g(x) = \sqrt{x}, \ x \geq 0$

(a) $f(x) \leq 4$ or $y \leq 4$

(b) $gf(x) = g[f(x)]$

$$= \sqrt{[f(x)]}$$

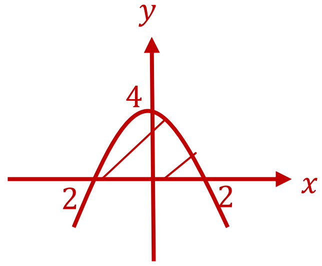
$$= \sqrt{4 - x^2}$$

Finding the domain:

$4 - x^2 \geq 0$

$(2-x)(2+x) \geq 0$

Critical values: $x = \pm 2$



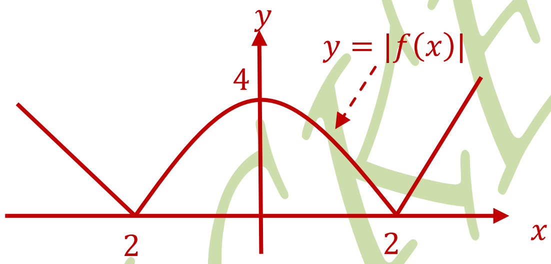
Since the inequality sign is ≥ 0 which means that the required solutions are those above the $x - axis$.

$$x \in \mathbb{R}: -2 \leq x \leq 2$$

$\therefore$ The domain of $gf(x) = x \in \mathbb{R}: -2 \leq x \leq 2$

(c) $f(x) = 4 - x^2,\ x \in \mathbb{R}$

When $x = 0$, $f(x) = 4$ and when $f(x) = 0$, $x = \pm 2$



(d) An injective function is a one-one function but since the domain of $f(x)$ is not restricted i.e. $x \in \mathbb{R}$ and also the function is not satisfying the horizontal line test $\therefore$ is not an injective function.

PRACTICE QUESTIONS

QUESTION 1

The functions f and g are defined by:

$$f(x) = \sqrt{2x+1}, \qquad x \in \mathbb{R}: x \geq -\frac{1}{2}$$

$$g(x) = 10x^2 - 9, \qquad x \in \mathbb{R}$$

(a) Find $g^{-1}(x)$.

(b) Solve the equation $g^{-1}(x) = f(x)$.

Answer: (a) $g^{-1}(x) = \sqrt{\frac{1}{10}(x+9)}$.

(b) $x = -\frac{1}{19}$

QUESTION 2

The function f is defined by

$$f(x) = \frac{3x+2}{5x+3}, \qquad x \in \mathbb{R}: x \neq -\frac{3}{5}$$

Solve the equation $f^{-1}(x) = x$.

Answer: $f^{-1}(x) = \frac{2-3x}{5x-3}$, $x \in \mathbb{R}: x \neq \frac{3}{5}$

$x = \pm\sqrt{\frac{2}{5}}$

QUESTION 3

The functions f and g are defined by:

$$f: x \rightarrow \frac{\sqrt{x-3}}{3}, \qquad x \in \mathbb{R}: x \geq 3$$

$$g: x \rightarrow 4x^2 + 3, \qquad x \in \mathbb{R}$$

(c) Find $fg(x)$.

(d) Hence state the relationship between the functions $f: x$ and $g: x$.

Answer: (a) $fg(x) = x$

(b) $f: x$ is the inverse of $g: x$

QUESTION 4

The functions f and g are defined by:

$$f: x \rightarrow \sqrt[3]{6-x}, \qquad x \in \mathbb{R}$$
$$g: x \rightarrow 5x^3, \qquad x \in \mathbb{R}$$

Solve the equation $f^{-1}(x) = g(x)$.

Answer: $f^{-1}(x) = 6 - x^3$

$x = 1$

QUESTION 5

Function f is defined by $f(x) = x^2 + 2x - 3,\ x \in \mathbb{R}$. Find $f^{-1}(x)$.

Answer: $f^{-1} = -1 + \sqrt{x+4};\ x \in \mathbb{R}: x \geq -4$

QUESTION 6

Given that $(x) = \frac{1-9x}{3x+6}$, describe geometrical transformations required to obtain the graph of $f(x)$ from the graph of $y = \frac{1}{x}$.

Answer: Stretch along the x-axis dividing every x coordinate by 3

Translation along the x-axis moving 6 units leftwards

Stretch along the y-axis multiplying every y coordinate by 19

Translation along the y-axis moving 3 units downwards

QUESTION 7

Find the domain of:

$$f(x) = \frac{x+2}{x^2+5x+4}?$$

Answer: $x \in \mathbb{R}: (x \neq -1)$ and $(x \neq -4)$

QUESTION 7

The functions f and g are defined by:

$$f(x) = x^2 + 3x + 1, \qquad x \in \mathbb{R}$$

$$g(x) = 2x - 3, \qquad x \in \mathbb{R}$$

Find $gf(x)$.

Answer: $gf(x) = 2x^2 + 6x - 1,\ x \in \mathbb{R}$

QUESTION 8

The functions f and g are defined by:

$$f(x) = 4x^2 - 5, \qquad x \in \mathbb{R}$$

$$g(x) = 2x - 3, \qquad x \in \mathbb{R}$$

Solve the equation $g \circ f = 787$.

Answer: $g \circ f = 8x^2 - 13$

$x = \pm 10$

QUESTION 9

The functions f and g are defined by:

$$f(x) = \frac{x}{x+1}, \qquad x \in \mathbb{R}: x \neq -1$$

$$g(x) = 2x - 1, \qquad x \in \mathbb{R}$$

Find

(i) $fg(x)$

(ii) a) $gf(x)$

b) $gf(3)$

Answer: (i) $fg(x) = \frac{2x-1}{2x}, x \in \mathbb{R}: x \neq 0$

(ii)(a) $gf(x) = \frac{x-1}{x+1}, x \in \mathbb{R}: x \neq -1$

(b) $gf(x) = \frac{1}{2}$

QUESTION 10

The functions f and g are defined by:

$$f(x) = 3x + 3, \qquad x \in \mathbb{R}$$

$$g(x) = 2x + 1, \qquad x \in \mathbb{R}$$

Find

(i) $g^{-1}(x)$

(ii) $fg^{-1}(9)$

(iii) Given that the point $A(2, 9)$ is on $f(x)$. State the coordinates of the corresponding point on $f^{-1}(x)$

Answer: (i) $g^{-1}(x) = \frac{x-1}{2}, x \in \mathbb{R}$

(ii) $fg^{-1}(9) = 18$

(iii) $A'(9, 2)$

QUESTION 11

The functions f and g are defined by:

$$f(x) = \frac{4}{2x+1}, \qquad x \in \mathbb{R}: x \neq -\frac{1}{2}$$

$$g(x) = \frac{a-x}{bx}, \qquad x \in \mathbb{R}: x \neq 0$$

If $fg(x) = x$, find a and b.

Answer: $fg(x) = x \Rightarrow g(x)$ is the inverse of $f(x)$

$\Rightarrow f^{-}(x) = \dfrac{4-x}{2x}$

$\Rightarrow a = 4$ and $b = 2$

QUESTION 12

The function f and g is defined by $f(x) = 3x - 5, x \in \mathbb{R}$. Solve $f(x^2) = 43$

Answer: $f(x^2) = 3x^2 - 5$

$\Rightarrow x = \pm 4$

QUESTION 13

The functions f and g are defined by:

$$f(x) = \sqrt{2x - 1} + 1, \qquad x \in \mathbb{R}: x \geq \frac{1}{2}$$

$$g(x) = kx - 3, \qquad x \in \mathbb{R}: x \neq 0$$

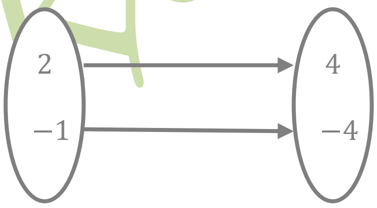
If the roots of the equation $2f^{-1}(x) = g(x)$ are equal find possible values of k.

Answer: $f^{-1}(x) = \frac{(x-1)^2+1}{2}$

$k = -2 \pm 2\sqrt{5}$

QUESTION 13

The diagram below represents 2 elements from the domain of the function $f(x) = \frac{12}{ax+b}$, where a and b are constants, together with their respective images.



(a) Find the values of a and b

(b) What is the domain of $f(x)$?

(c) Find the inverse function, $f^{-1}(x)$

(d) Hence, or otherwise, find the element that under f has an image $\frac{4}{5}$.

Answer: (a) $a = 2$ and $b = -1$ (b) $f(x) = \frac{12}{2x-1}$

(c) $f^{-1}(x) = \frac{12+x}{2x}$ (d) $f^{-1}\left(\frac{4}{5}\right) = 8$

PAST EXAMINATION QUESTIONS

UCLES NOVEMBER 2000 PAPER 1

The function f is defined by

$$f : x \to \frac{x}{x+1}, \qquad x \in \Re, \qquad x \neq 1$$

(i) Express $f^{-1}(x)$ in terms of x, **[3]**

(ii) Verify that $f(x)$ may be written in the form

$1 - \frac{1}{x+1}$ **[1]**

Hence identify a sequence of transformations under which the graph of $y = \frac{1}{x}$ is transformed to that of $y = f(x)$ **[3]**

Answer: (i) $f^{-1}(x) = \frac{x}{1-x}$, $x \in \Re,\ x \neq 1$

(ii) Translation along the x-axis moving 1 unit leftwards
Reflection along the x-axis
Translation along the y-axis moving 1 unit upwards

ZIMSEC NOVEMBER 2004 PAPER 1

Given that the functions

$$f : x \to 4x - 10 \text{ and}$$

$$h : x \to \frac{8}{x}, x \neq 0$$

express,

a) in a similar form

(i) $f \circ g$, **[1]**

(ii) $g \circ f$, **[1]**

(iii) g^{15} [Hint $g^3 = g \circ g \circ g$. **[2]**

b) in term of one or both of f and g

(i) $x \to \frac{1}{4}(x + 10)$ **[1]**

(ii) $x \to 16x - 50$ **[1]**

Answer: a) (i) $f \circ g = \frac{32}{x} - 10\,,\ x \in \Re,\ x \neq 0$

(ii) $g \circ f = \frac{4}{2x-5}\,,\ x \in \Re,\ x \neq \frac{5}{2}$

(iii) $g^{15} = \frac{x}{1-x}\,,\ x \in \Re,\ x \neq 1$

b) (i) $f^{-1}: x = \frac{1}{4}(x+10),\ x \in \Re$

(ii) $f \circ f = 16x - 50$

ZIMSEC JUNE 2005 PAPER 1

Given that $f(x) = e^{2x-3}$ for all real values of x, find an expression for $f^{-1}(x)$, stating the domain for $f^{-1}(x)$ clearly. **[4]**

Answer: $f^{-1}(x) = \frac{Inx+3}{2},\ x \in \Re,\ x > 0$

ZIMSEC NOVEMBER 2007 PAPER 1

A function is defined by

$$f: x \to x^2 + 4x + 1, \text{for } x \geq -2$$

Find

(i) the range of the function, **[2]**

(ii) an expression for $f^{-1}(x)$, stating its domain. **[4]**

Answer: (i) $f(x) \geq -3$

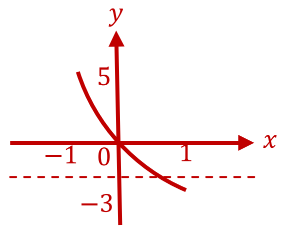
(ii) $f^{-1}(x) = \sqrt{x+3} - 2, x \in \Re,\ x \geq -3$

ZIMSEC JUNE 2009 PAPER 1

The function f is defined by $f: x \to x^2 - 4x,\ x \in \Re,\ |x| \leq 1$

Show by means of a graphical argument or otherwise, that f is one$-$one, and find an expressions for $f^{-1}(x)$. **[5]**

Answer:

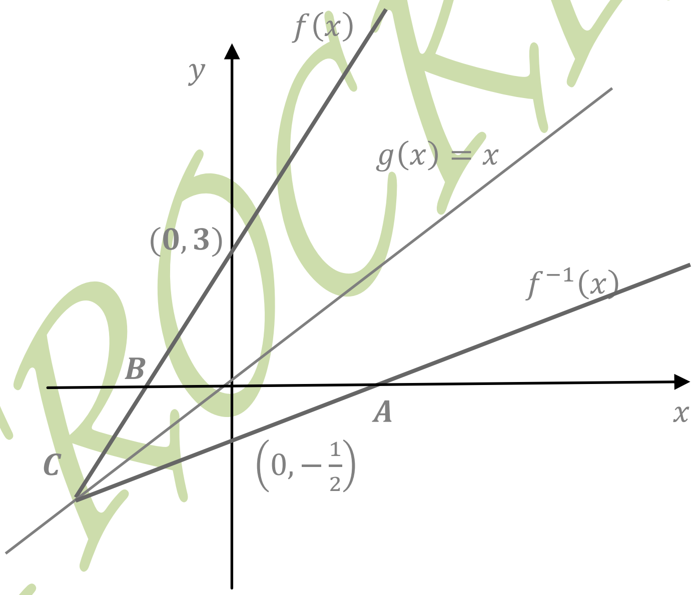


The horizontal line cuts the graph once ∴ it is a one to one function

$f^{-1}(x) = \sqrt{x+4} + 2\ ,\ \ x \in \Re, -3 \leq x \leq 5$

ZIMSEC NOVEMEBR 2010 PAPER 1

The diagram below shows the linear graphs of $f(x), g(x)$ and $f^{-1}(x)$ which intersect at $\boldsymbol{C}$. The graph of $f^{-1}(x)$ intersects the $x - axis$ at point $\boldsymbol{A}$ and $f(x)$ intersects the $x - axis$ at $\boldsymbol{B}$.



a) State the name given to the function $g(x)$ in relation to the functions $f(x)$ and $f^{-1}(x)$. **[1]**

b) Write down the co-ordinates of $\boldsymbol{A}$ and $\boldsymbol{B}$. **[2]**

c) Calculate the coordinates of the point $\boldsymbol{C}$. **[4]**

Answer: (a) mirror line

(b) $A(3,\ \ 0)$ $\quad B\left(-1\frac{1}{2},\ \ 0\right)$

(c) $C(-3,-3)$

ZIMSEC NOVEMBER 2011 PAPER 1

The function $h(x) = 2x^2 - 6x + 11,\ x \in \Re$

Express

(a) $h(x)$ in the form $a(x+b)^2 + c$ where a, b and c are constants. **[2]**

(b) Write down the range of h. **[1]**

(c) Explain why h does not have an inverse. **[1]**

(d) Given that $h(x)$ is defined for $x \in A$, where $A \subset \Re$, find

(i) the largest element of A for which $h(x)$ has an inverse,

(ii) $h^{-1}(x)$ for this set of elements of A. **[4]**

Answer: (a) $2\left(x - \frac{3}{2}\right)^2 + \frac{13}{2}$

(b) $h(x) \geq \frac{13}{2}$

(c) $h(x$ is not a one-one function

(d) (i) $A = \frac{3}{2}$

(ii) $h^{-1}(x) = \frac{3-\sqrt{2x-13}}{2}$, $x \in \Re,\ x \geq \frac{13}{2}$

ZIMSEC JUNE 2014 PAPER 1

The functions f and g are defined for $x \in \Re$ by

$f: x \longrightarrow x^2 + 4x + 1$

$g: x \longrightarrow ax + b$

Given that $fg(2) = -2$ and $gf(0) = -3$, find the values of a and b. **[5]**

Answer: $a = 0$ *or* 2 and $b = -3$ *or* -5

ZIMSEC NOVEMBER 2016 PAPER 1

The function f is defined by $f(x) = 2x - 3$, for $x \in \Re$

(a) Sketch on a single diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$ making clear relationships between the graphs. **[2]**

(b) (i) The function g is defined by $g(x) = 3x^2 + 2x - 2$, for $x \in \Re$.

Express $gf(x)$ in terms of x in its simplest form. **[2]**

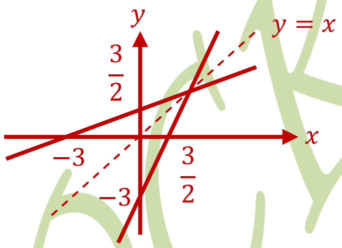
(ii) Hence find the minimum value of $gf(x)$. **[3]**

(c) Given that $g(x) = 3x^2 + 2x - 2$ is defined for $x \geq k$,

(i) state the minimum value of k, such that $g(x)$ can have an inverse, **[3]**

(ii) find $g^{-1}(x)$. **[2]**

Answer: (a)



The graphs of $y = f(x)$ and $y = f^{-1}(x)$ are reflecting on the line $y = x$

(b)(i) $gf(x) =$

(ii) The minimum value of $gf(x) =$

(c)(i) The minimum value of $k =$

(ii) $f^{-1}(x) = \frac{x}{1-x}$, $x \in \Re$, $x \neq 1$

ZIMSEC NOVEMBER 2017 PAPER 1

Functions f and h are defined as

$f(x) = 3x - 1, x \in \Re,$

$h(x) = 2x + 5, x \in \Re$

Find the value of x for which $fh(x) = 2hf(x)$. **[4]**

Answer: $x = \frac{4}{3}$

ZIMSEC JUNE 2018 PAPER 1

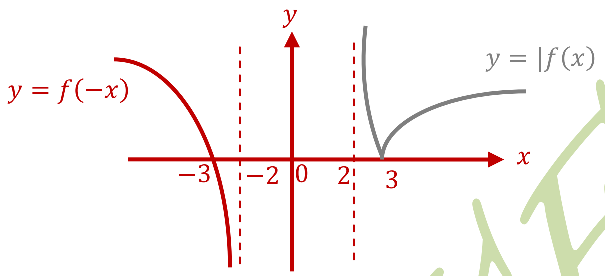
The function $f(x)$ is defined by $f(x) = In(x-2)$

(a) State the domain of $f(x)$. **[1]**

(b) On the same axes, sketch the graphs of $f(-x)$ and $y = |f(x)|$. **[4]**

Answer: (a) $x \in \Re,\ x > 2$

(b)



ZIMSEC NOVEMBER 2018 PAPER 1

The function $f(x) = -x^2 + 8x - 12$ is defined for $4 \leq x \leq 6$.

(i) Express $f(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. **[2]**

(ii) State why the function of $f(x)$ has an inverse. **[1]**

(iii)Find $f^{-1}(x)$, the inverse of $f(x)$. **[2]**

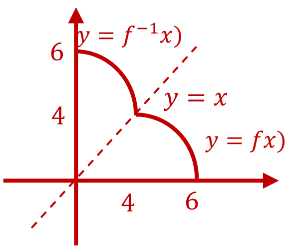
(iv)Sketch the graphs of $f(x)$ and $f^{-1}(x)$ on the same axes and write down the domain of $f^{-1}(x)$. **[3]**

Answer: (i)$-(x-4)^2 + 4$

(ii) $f(x)$ is a one-one function/ an injective function

(iii) $f^{-1}(x) = 4 + \sqrt{4-x}\ ,\ x \in \Re: 0 \leq x \leq 4$

(iv)



Domain of $f^{-1}(x) = x \in \Re:\ 0 \leq x \leq 4$

NB: Domain of $f^{-1}(x)$ = Range of $f(x)$

ZIMSEC JUNE 2019 PAPER 1

The functions f and h are defined as

$$f: x \to e^x, x \in \Re,$$

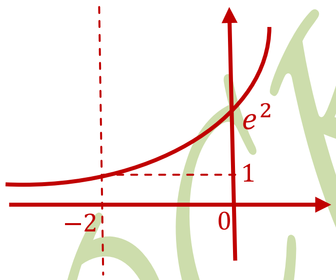
$$h: x \to x + 2, x \in \Re.$$

(a) Sketch the graph of $fh(x)$. **[2]**

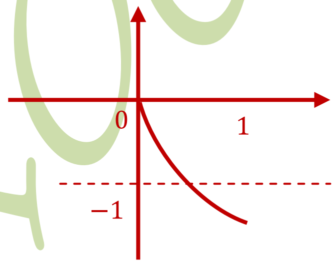
(b) (i) Show by graphical argument or otherwise that $g: x \to x^2 - 2x$, $x \in \Re, 0 \leq x \leq 1$ is one to one. **[2]**

(ii) Find an expression for $g^{-1}(x)$. **[2]**

Answer: (a) $fh(x) = e^{x+2}$



(b)(i)



The horizontal line cuts the graph once ∴ it is a one to one function

(ii) $g^{-1}(x) = \sqrt{x+1} - 1$, $x \in \Re, -1 \leq x \leq 0$

ZIMSEC JUNE 2019 PAPER 2

The function f is defined by $f(x) = In(2x - 3)$ for $x \in \mathbb{R}, x > \frac{3}{2}$.

(i) Find $f^{-1}(x)$ and state its domain. **[3]**

(ii) Solve the equation $f^{-1}(x) - 6f^{-1}(-x) + 5 = 0$. **[6]**

Answer: (i) $f^{-1}(x) = \frac{e^x+3}{12}$, $x \in \Re$

(ii) $x = In6$ or 1.8

ZIMSEC NOVEMBER 2019 PAPER 1

(a) Functions f and g are defined by:

$f: x \to Inx, x > 0,$

$g: x \to (x + 2), x > -2.$

Find

(i) $fg(x)$, **[1]**

(ii) the inverse of $fg(x)$. **[2]**

(b) Given that $h(\theta) = ksin\theta; k < 0$.

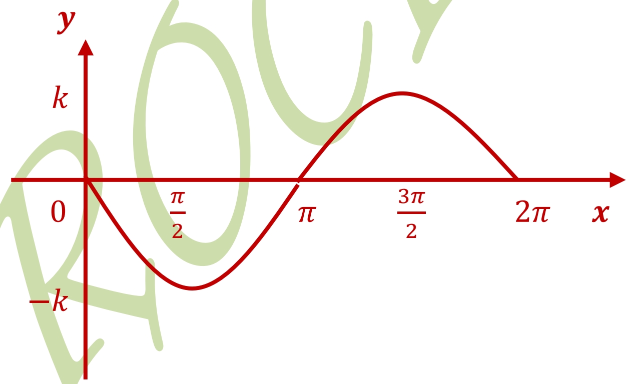
Sketch the graph of $y = h(\theta)$ for $0 \le \theta \le 2\pi$. **[3]**

(c) Describe the geometrical transformations which maps $f(x) = Sinx$ onto $y = 6Sin(x - \pi)$. **[3]**

Answer: a(i) $fg(x) = In(x + 2),\ x > -2$

(ii) $(fg)^{-1} = e^x - 2,\ x \in \Re$

b)



(c) Translation along the $x - axis$ with translation vector $\begin{pmatrix}\pi\\0\end{pmatrix}$ or moving π $units$ rightwards followed by a stretch along the $y - axis$ with factor 6 or multiply every y –coordinate by 6

ZIMSEC NOVEMBER 2019 PAPER 2

(a) Express $g(x) = \frac{2x-1}{x-3}$ in the form $g(x) = a + \frac{b}{x-3}$. **[2]**

(b) Hence state the correct sequence of transformation which transform the graph of $y = \frac{1}{x}$ onto the graph $y = g(x)$. **[4]**

Answer: (a) $g(x) = 2 + \frac{5}{x-3}$

(b) Translation along the x-axis moving 3 unit rightwards
Stretch along the y-axis with factor 5
Translation along the y-axis moving 2 unit upwards

ASANTE SANA

*******THERE IS A LIGHT AT THE END OF EVERY TUNNEL *******

CONSTRUCTIVE COMMENTS ON THE FORM OF THE PRESENTATION, INCLUDING ANY OMISSIONS OR ERRORS, ARE WELCOME.

ENJOY

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