Inequalities

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SYLLABUS (6042) REQUIREMENTS

- Solving problems involving inequalities (including rational inequalities)
- Modelling situations involving equations and inequalities and solving related problems

INEQUALITIES

Definition: These are expressions involving the signs $<, \leq, >, \geq$ or $\neq$. An inequality is just a statement that of two quantities one is specifically less than (or greater than) another.

Rules for solving inequalities

Suppose a, b and c are any given real numbers.

(i) If $a > b$ then $a + c > b + c$.
That is, adding (or subtracting) a number to both sides of an inequality does **not** change the direction of the inequality symbol.

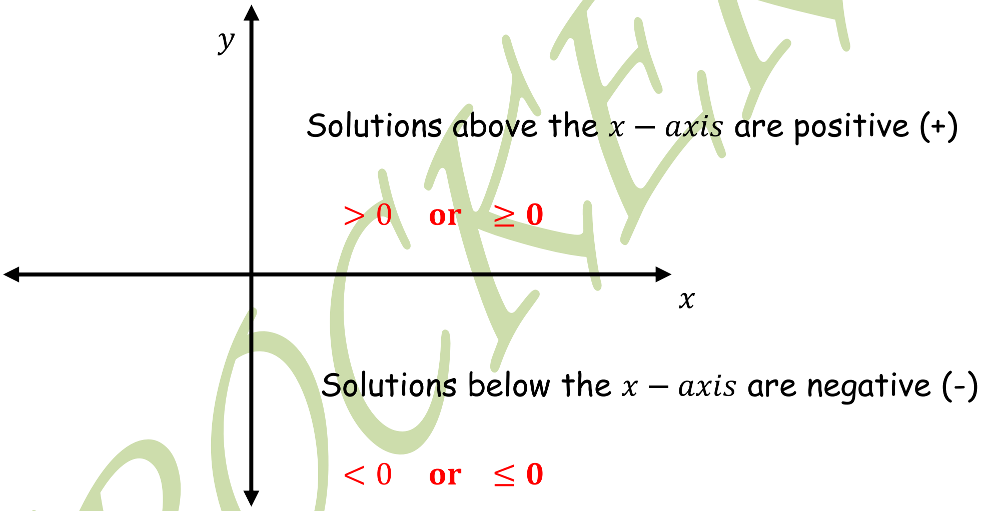
(ii) If $a > b$ and $c > 0$ then $ac > bc$.
That is, multiplying (or dividing) an inequality by a **positive** number does **not** change the direction of the inequality symbol.

(iii) If $a > b$ and $c < 0$ then $ac < bc$. That is, multiplying (or dividing) an inequality by a **negative** number **changes** the direction of the inequality symbol.

Quadratic and Cubic inequalities

STEPS

- Bring all functions to one side of the inequality
- Factorise the expression, where possible.
- Find all real zeros of the polynomial, and arrange the zeros in increasing order. The zeros of a polynomial are its critical numbers.
- Use the critical numbers to determine the test intervals.
- Use a table or graph to find the required solution. BUT the best method is graphical.



NOTE

When solving quadratic/cubic/higher order inequalities you need to consider the **critical values**. You calculate $f(x)$ in each of the regions of the number line created by the critical values and produce a sign diagram.

Solved Problems

Question 1

Solve the inequality $x^2 + 3x - 4 < 0$

Solution

$x^2 + 3x - 4 < 0$

$x^2 + 4x - 1x - 4 < 0$

$x(x + 4) - 1(x + 4) < 0$

$(x + 4)(x - 1) < 0$

The critical values are $x = -4$ and $x = 1$.

NB: To find the critical values we set each expression to zero as follows:

$x + 4 = 0$ and $x - 1 = 0 \Longrightarrow x = -4$ and $x = 1$.

The solution is determined using either the table or graphical method as follows:

Method 1: Table

	$x < -4$	$-4 < x < 1$	$x > 1$
$x + 4$	$-$	$+$	$+$
$x - 1$	$-$	$-$	$+$
$f(x)$	$+$	$-$	$+$

The solution set is $\{x \in \Re: -4 < x < 1\}$

Method 2: Graph

$(x + 4)(x - 1) < 0$

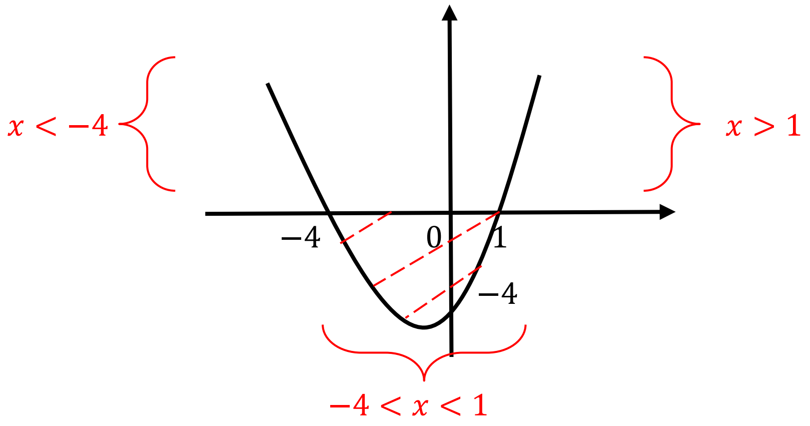
Considering the function $f(x) = (x + 4)(x - 1)$:

The graph cuts the $y - axis$ when $x = 0$ and $x - axis$ when $y = 0$.

Now:

When $x = 0$: $f(x) = (0 + 4)(0 - 1) = -4$ and when $f(x) = 0$: $0 = (x + 4)(x - 1)$

$\Rightarrow x = -4$ or 1 $(Critical\ Values)$.



Since the inequality sign is < 0 which means that the required solutions are those below the $x - axis$.

$\therefore$ The solution set is $\{x \in \Re: -4 < x < 1\}$.

Question 2

Solve the inequality $2x^2 - 4x > 3x - 5$

Solution

$2x^2 - 4x > 3x - 5$

$2x^2 - 4x - 3x + 5 > 0$

$2x^2 - 7x + 5 > 0$

$2x^2 - 5x - 2x + 5 > 0$

$x(2x - 5) - 1(2x - 5) > 0$

$(2x - 5)(x - 1) > 0$

The critical values are $x = 1$ and $x = \frac{5}{2}$.

Method 1: Table

	$x < 1$	$1 < x < \frac{5}{2}$	$x > \frac{5}{2}$
$2x - 5$	$-$	$-$	$+$
$x - 1$	$-$	$+$	$+$
$f(x)$	$+$	$-$	$+$

The solution set is $\left\{x \in \Re : x < 1 \text{ or } x > \frac{5}{2}\right\}$

Method 2: Graph

$(2x-5)(x-1) > 0$

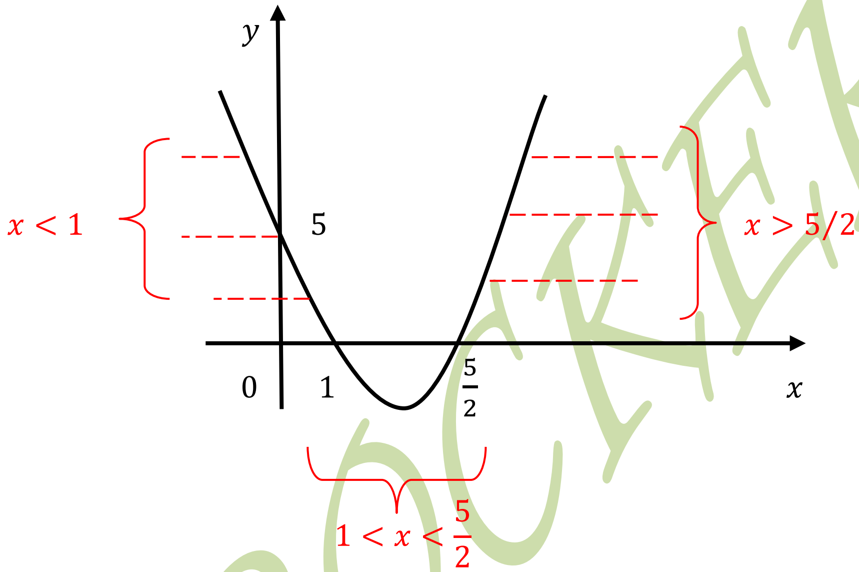
Considering the function $f(x) = (2x-5)(x-1)$:

The graph cuts the $y - axis$ when $x = 0$ and $x - axis$ when $y = 0$.

Now:

When $x = 0$: $f(x) = (0-5)(0-1) = 5$ and when $f(x) = 0$: $0 = (2x-5)(x-1)$

$\Rightarrow x = 1$ or $x = \frac{5}{2}$. $(Critical\ Values)$.



Since the inequality sign is > 0 which means that the required solutions are those above the $x - axis$.

$\therefore$ The solution set is $\left\{x \in \Re : x < 1 \text{ or } x > \frac{5}{2}\right\}$

Question 3

Solve the inequality $x(x-1)(x+3) \geq 0$

Solution

$x(x-1)(x+3) \geq 0$

The critical values are $x = 0$,$x = 1$ and $x = -3$

Method a: Table

	$x \leq -3$	$-3 \leq x \leq 0$	$0 \leq x \leq 1$	$x \geq 1$
x	$-$	$-$	$+$	$+$
$x-1$	$-$	$-$	$-$	$+$
$x+3$	$-$	$+$	$+$	$+$
$f(x)$	$-$	$+$	$-$	$+$

The solution set is $\{x \in \Re: -3 \leq x \leq 0 \text{ or } x \geq 1\}$

Method b: Graph

$x(x-1)(x+3) \geq 0$

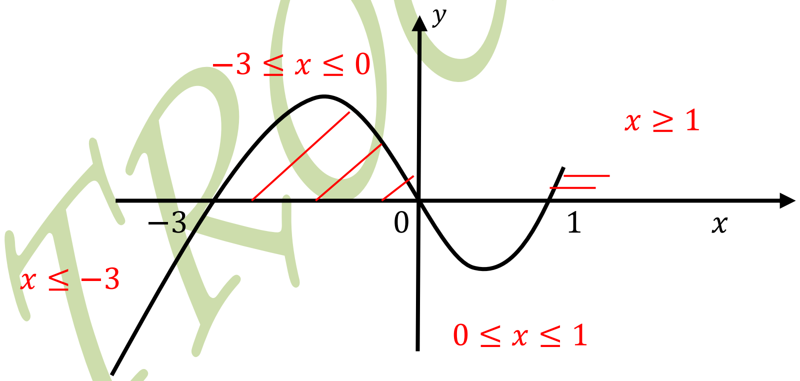
Let's consider the function $f(x) = x(x-1)(x+3)$

The graph cuts the $y-axis$ when $x = 0$ and $x-axis$ when $y = 0$.

Now:

When $x = 0$: $f(x) = 0(0-1)(0+3) = 0$ and when $f(x) = 0$: $0 = x(x-1)(x+3)$

$\Rightarrow x = 0, 1$ or $x = -3(Critical\ Values)$.



Since the inequality sign is ≥ 0 which means that the required solutions are those above the $x-axis$.

$\therefore$ The solution set is $\{x \in \Re: -3 \leq x \leq 0 \text{ or } x \geq 1\}$

Follow up Exercise

Question 1

Solve the inequality $x^2 - x \geq 12$

Answer: $\{x \in \Re:\ x \leq -3 \text{ or } x \geq 4\}$

Question 2

Solve the inequality $x^2 > 6 - x$

Answer: $\{x \in \Re:\ x < -3 \text{ or } x > 2\}$

Question 3

Solve the inequality $2x^2 + 5x \geq 12$

Answer: $\{x \in \Re: x \leq -4 \text{ or } x \geq 3/2\}$

Question 4

Solve the inequality $3x^2 < -10 - 13x$

Answer: $\{x \in \Re:\ -10/3 < x < -1\}$

Question 5

Solve the inequality $x^2 + 3x - 18 \geq 0$

Answer: $\{x \in \Re:\ x \leq -6 \text{ or } x \geq 3\}$

Question 6

Solve the inequality $2x^2 - x < 3$

Answer: $\{x \in \Re:\ -1 < x < 3/2\}$

Question 7

Solve the inequality $x^3 - x \leq 3x^2 - 3$

Answer: $\{x \in \Re: x \leq -1 \text{ or } 1 \leq x \leq 3\}$

Question 8

Solve the inequality $x^3 - 2x^2 \leq 5x - 6$

Answer: $\{x \in \Re: x \leq -2 \text{ or } 1 \leq x \leq 3\}$

Question 9

Solve the inequality $x^3 + 4x^2 - 11x - 30 < 0$

Answer: $\{x \in \Re: x < -5 \text{ or } -2 < x < 3\}$

Question 10

Solve the inequality $(x+2)(x-3)(x-7) > 0$

Answer: $\{x \in \Re: -2 < x < 3 \text{ or } x > 7\}$

Inequalities involving rational fractions

- When solving inequalities involving a rational expression, you must ensure that the expression is written in an appropriate form (i.e. factorised/simplified as far as possible and with 0 on the right-hand side). You can then find the critical values for the numerator and denominator.
- Never cross-multiply unless the denominator is positive definite i.e.
- Examples of positive definite expressions:
  $ax^2 + b$ where $(a, b) > 0$ or $(ax+b)^2$ or $(ax-b)^2$

Solved Problems

Question 1

Solve the inequality

$$\frac{x-1}{3-x} < 2$$

Suggested Solution

Method 1

$$\frac{x-1}{3-x} < 2$$

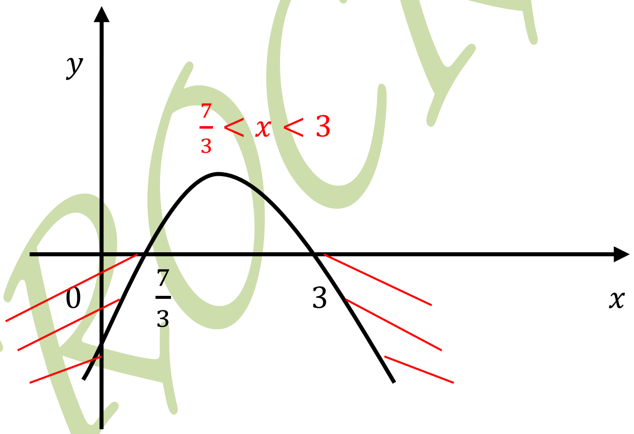
$$\frac{x-1}{3-x} - 2 < 0$$

$$\frac{x-1-2(3-x)}{3-x} < 0$$

$$\frac{x-1-6+2x}{3-x} < 0$$

$$\frac{3x-7}{3-x} < 0$$

The critical values of x are: $x = 3$ and $= \frac{7}{3}$.



Since the inequality sign is < 0 which means that the required solutions are those below the $x - axis$.

$\therefore$ The solution set is $\left\{x \in \Re : x < \frac{7}{3} \text{ or } x > 3\right\}$

Method 2

$$\frac{x-1}{3-x} < 2$$

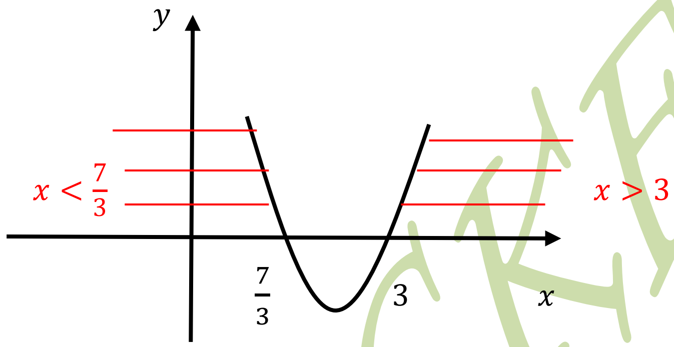
$$(3-x)^2\frac{x-1}{3-x} < 2(3-x)^2$$

$$\Rightarrow (3-x)(x-1) < 2((9-6x+x^2)$$

$$3x^2-16x+21 > 0$$

$$(3x-7)(x-3) > 0$$

The critical values of x are: $x = 3$ and $= \frac{7}{3}$.



Since the inequality sign is > 0 which means that the required solutions are those above the $x - axis$.

$\therefore$ The solution set is $\left\{x \in \Re : x < \frac{7}{3} \text{ or } x > 3\right\}$

<u>Question 2</u>

<u>ZIMSEC NOVEMBER 2013 PAPER 1</u>

Solve the inequality

$$\frac{3x+1}{9-x^2} \geq -1$$ **[4]**

<u>Suggested Solution</u>

$$\frac{3x+1}{9-x^2} \geq -1$$

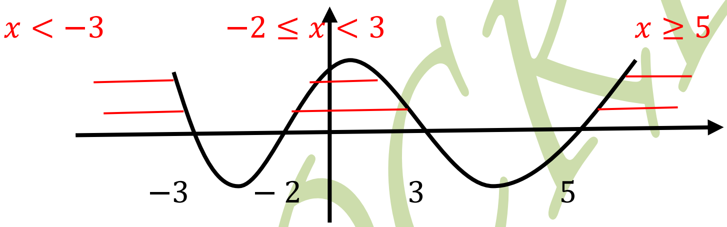
$$\frac{3x+1}{9-x^2} + 1 \geq 0$$

$$\frac{3x+1+1(9-x^2)}{9-x^2} \geq 0$$

$$\frac{10+3x-x^2}{9-x^2} \geq 0$$

$$\frac{(2+x)(5-x)}{(3-x)(3+x)} \geq 0$$

The critical values of x are: $x = -2$; $x = -3$; $x = 3$ and $x = 5$.



Since the inequality sign is ≥ 0 which means that the required solutions are those above the $x - axis$.

$\therefore$ The solution set is $\{x \in \Re : x < -3 \text{ or } -2 \leq x < 3 \quad \text{or } x \geq 5\}$

NB: For the example above; 3 and -3 must not be part of the solution because they make the fraction $\frac{(2+x)(5-x)}{(3-x)(3+x)}$ undefined.

Question 3

ZIMSEC NOVEMBER 2018 PAPER 1

Solve the inequality

$$\frac{21 + 13x}{2 + 3x^2} \geq 10$$ **[4]**

Suggested Solution

Note: Since the denominator is positive definite, we should multiply both sides by

$2 + 3x^2$.

$$\frac{21 + 13x}{2 + 3x^2} \geq 10$$

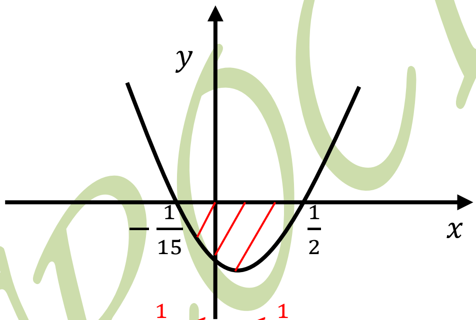
$$21 + 13x \geq 10(2 + 3x^2)$$

$$\Rightarrow 30x^2 - 13x - 1 \leq 0$$

$$\Rightarrow 30x^2 15x + 2x - 1 \leq 0$$

$$\Rightarrow (15x + 1)(2x - 1) \leq 0$$

The critical values of x are: $x = -\frac{1}{15}$ and $x = \frac{1}{2}$.



$$-\frac{1}{15} \leq x \leq \frac{1}{2}$$

Since the inequality sign is < 0 which means that the required solutions are those below the $x - axis$.

$\therefore$ The solution set is $\left\{x \in \Re: \quad -\frac{1}{15} \leq x \leq \frac{1}{2}\right\}$

Question 4

ZIMSEC NOVEMBER 2018 PAPER 2

Solve the inequality

$$\frac{x-2}{x+1} < \frac{x-6}{x-2} \qquad [\mathbf{6}]$$

Suggested Solution

$$\frac{x-2}{x+1} < \frac{x-6}{x-2}$$

$$\frac{x-2}{x+1} - \frac{x-6}{x-2} < 0$$

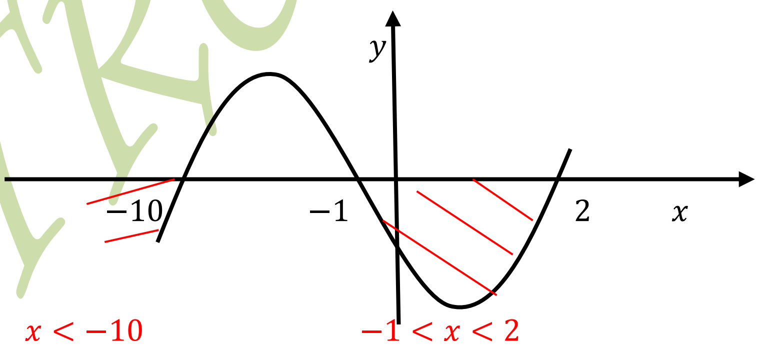
$$\frac{(x-2)(x-2)-(x-6)(x+1)}{(x+1)(x-2)} < 0$$

$$\frac{x^2-4x+4-[x^2-5x-6]}{(x+1)(x-2)} < 0$$

$$\frac{x^2-4x+4-x^2+5x+6}{(x+1)(x-2)} < 0$$

$$\frac{x+10}{(x+1)(x-2)} < 0$$

The critical values of x are: $x=-10$; $x=-1$ and $x=2$.



Since the inequality sign is <0 which means that the required solutions are those below the $x-axis$.

$\therefore$ The solution set is $\{x \in \Re : x < -10 \; or -1 < x < 2\}$

Question 5

Find the solution set of the inequality $\frac{12}{x-3} < x + 1$.

$$\frac{12}{x-3} < x + 1$$

$$\Rightarrow \frac{12}{x-3} - (x+1) < 0$$

$$\Rightarrow \frac{12-(x+1)(x-3)}{x-3} < 0$$

$$\Rightarrow \frac{12-(x^2-3x+x-3)}{x-3} < 0$$

$$\Rightarrow \frac{12-(x^2-2x-3)}{x-3} < 0$$

$$\Rightarrow \frac{12-x^2+2x+3}{x-3} < 0$$

$$\Rightarrow \frac{15+2x-x^2}{x-3} < 0$$

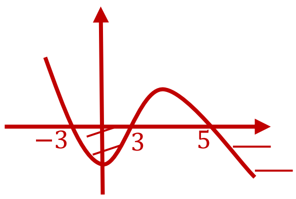
Factorising$15 + 2x - x^2$:

$$\Rightarrow \frac{15+5x-3x-x^2}{x-3} < 0$$

$$\Rightarrow \frac{5(3+x)-x(3+x)}{x-3} < 0$$

$$\Rightarrow \frac{(5-x)(3+x)}{x-3} < 0$$

∴The critical values are $x = 5, x = 3$ and $x = -3$



Since the inequality sign is < 0 which means that the required solutions are those below the x-$axis$.

∴ The solution set is $\{x \in \Re: -3 < x < 3 \;\; or \;\; x > 5\}$

Follow up Exercise

Question 1

Solve the inequality $\frac{x^2+x-6}{x-4} \geq 0$

Answer: $\{x \in \Re:\ -3 \leq x \leq 2 \text{ or } x > 4\}$

Question 2

Solve the inequality $\frac{x-5}{x-8} \geq\ 3$

Answer: $\{x \in \Re:\ 8 < x \leq 19/2\}$

Question 3

Solve the inequality $\frac{-2}{1-x} < 7$

Answer: $\{x \in \Re:\ x < 1\ or\ x > 9/7\ \}$

Question 4

Solve the inequality $\frac{x-6}{x+2} < -1$

Answer: $\{x \in \Re:\ -2 < x < 2\}$

Question 5

Solve the inequality $\frac{5}{x-3} \geq \frac{4}{x-2}$

Answer: $\{x \in \Re:\ -2 \leq x < 2 \text{ or } x > 3\}$

Question 6

Solve the inequality $\frac{x-5}{2x+1} < \frac{4x}{2x+1} - 3$

Answer: $\{x \in \Re\colon -1/2 < x < 2/3\}$

Question 7

Solve the inequality $\frac{3x+5}{x+1} \leq 2$

Answer: $\{x \in \Re\colon -3 \leq x < 1\}$

Question 8

Solve the inequality $\frac{x+2}{x-1} \leq 3$

Answer: $\{x \in \Re\colon x < 1 \text{ or } x \geq 5/2\}$

Question 9

Solve the inequality $\frac{2x-3}{x^2+1} \geq 0$

Answer: $\{x \in \Re\colon x \geq 3/2\}$

Question 10

Solve the inequality $\frac{x^2-9}{2x-1} \leq 0$

Answer: $\{x \in \Re\colon x \leq -3 \text{ or } 1/2 < x \leq 3\}$

Question 11

Solve the inequality $\frac{3x+4}{x+1} \leq 2$

Answer: $\{x \in \Re: -2 \leq x < -1\}$

Question 12

Solve the inequality $\frac{5}{x-3} \geq \frac{4}{x-2}$

Answer: $\{x \in \Re: -2 \leq x < 2 \text{ or } x > 3\}$

Absolute Values

The absolute value of x, denoted by $|x|$, is the distance from x to 0 on the number line. Since distances are always positive or 0, we have $|x| \geq 0$

In general, we have

$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

Absolute values can also be removed by using the following formula:

$$|x| = \sqrt{x^2}$$

In general, for any positive number a, we have $|x| = a$ if and only if

$$x = \begin{cases} a \\ -a \end{cases}$$

NOTE

Solving the equation $|x| = 2$ leads to two answers: $x = \pm 2$.

However, finding $|2|$ leads to only one answer, namely $|2| = 2$.

Inequalities involving Modulus signs

Case 1

$|x \pm a| < b$ where b is any positive constant.

Worked Problem

Example

Solve the inequality $|x - 2| < 5$

Suggested Solution

Method 1

$|x - 2| < 5$

$-5 < x - 2 < 5$

$-5 + 2 < x < 5 + 2$

$\therefore$The solution set is $\{x \in \Re: -3 < x < 7\}$

Method 2

Squaring both sides:

$|x-2| < 5$

$(x-2)^2 < 5^2$

$x^2 - 4x + 4 < 25$

$x^2 - 4x - 21 < 0$

$x^2 - 7x + 3x - 21 < 0$

$x(x-7) + 3(x-7) < 0$

$(x+3)(x-7) < 0$

The critical values are: $x = -3$ and $x = 7$

Method a: Table

	$x < -3$	$-3 < x < 7$	$x > 7$
$x+3$	$-$	$+$	$+$
$x-7$	$-$	$-$	$+$
$f(x)$	$+$	$-$	$+$

The solution set is $\{x \in \Re : -3 < x < 7\}$

Method b: Graph

$(x+3)(x-7) < 0$

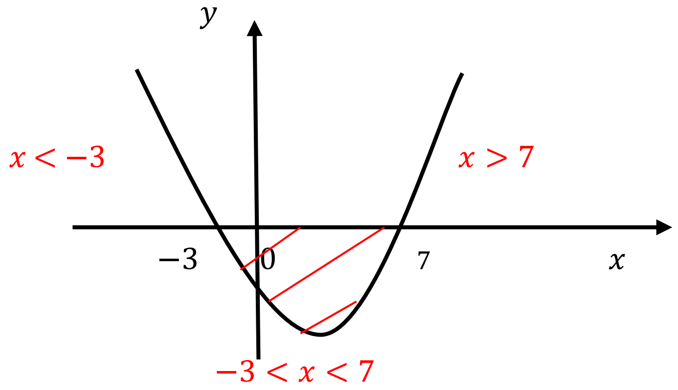
Considering the function $f(x) = (x+3)(x-7)$:

The graph cuts the $y - axis$ when $x = 0$ and $x - axis$ when $y = 0$.

Now:

When $x = 0$: $f(x) = (0+3)(0-7) = -21$ and when $f(x) = 0$: $0 = (x+3)(x-7)$

$\Rightarrow x = -3$ or $x = 7. (Critical\ Values).$



Since the inequality sign is < 0 which means that the required solutions are those below the $x - axis$.

$\therefore$ The solution set is $\{x \in \Re: -3 < x < 7\}$

Method 3

Graphical method

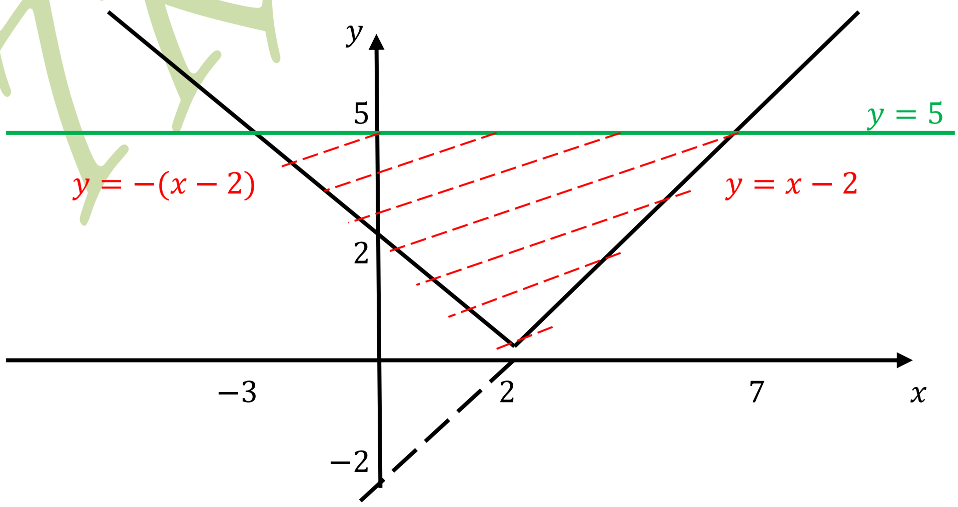
$|x - 2| < 5$

LHS

$y = x - 2$: When $x = 0$; $y = -2$ and when $y = 0$; $x = 2$

RHS

$y = 5$.



From the diagram above we calculate the critical values as follows:

$-(x-2) = 5$ and $x - 2 = 5$

$-x + 2 = 5$ $\quad$ $x = 5 + 2$

$x = -3$ $\quad$ $x = 7$

Now to solve $|x-2| < 5$ we consider the shaded part.

$\therefore$The solution set is $\{x \in \Re : -3 < x < 7\}$

Case 2

$|x \pm a| > b$ where b is any positive constant.

Method 1

$|x \pm a| < b$ gives $x \pm a > b$ or $x \pm a < -b$

Method 2

Squaring both sides:

$|x \pm a| > b$ gives $(x \pm a)^2 > b^2$

Method 3

Graphical method

Solved Problem

Question

Solve the inequality $|x+1| \geq 5$

Suggested Solution

Method 1

$|x + 1| \geq 5$

$x + 1 \leq -5$ or $x + 1 \geq 5$

$x \leq -5 - 1$ or $x \geq 5 - 1$

$\therefore$The solution set is $\{x \in \Re: x \leq -6 \text{ or } x \geq 4\}$

Method 2

Squaring both sides:

$|x + 1| \geq 5$

$(x + 1)^2 \geq 5^2$

$x^2 + 2x + 1 \geq 25$

$x^2 + 2x - 24 \geq 0$

$x^2 + 6x - 4x - 24 \geq 0$

$x(x + 6) - 4(x + 6) \geq 0$

$(x + 6)(x - 4) \geq 0$

The critical values are: $x = -6$ and $x = 4$

Method a: Table

	$x \leq -6$	$-6 \leq x \leq 4$	$x \geq 4$
$x + 6$	$-$	$+$	$+$
$x - 4$	$-$	$-$	$+$
$f(x)$	$+$	$-$	$+$

$\therefore$The solution set is $\{x \in \Re: x \leq -6 \quad \text{or} \quad x \geq 4\}$

Method b: Graph

$(x+6)(x-4) \geq 0$

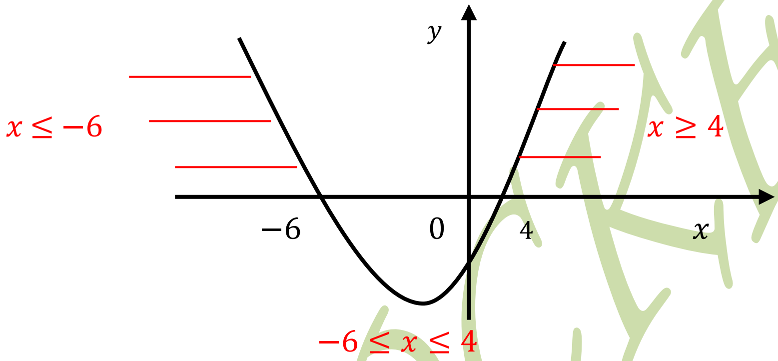
Considering the function $f(x) = (x+6)(x-4)$:

The graph cuts the $y - axis$ when $x = 0$ and $x - axis$ when $y = 0$.

Now:

When $x = 0$: $f(x) = (0+6)(0-4) = -24$ and when $f(x) = 0$: $0 = (x+6)(x-4)$

$\Rightarrow x = -6$ or $x = 4$. $(Critical\ Values)$.



Since the inequality sign is ≥ 0 which means that the required solutions are those above the $x - axis$.

$\therefore$The solution set is $\{x \in \Re: x \leq -6 \quad \text{or} \quad x \geq 4\}$

Method 3

Graphical method:

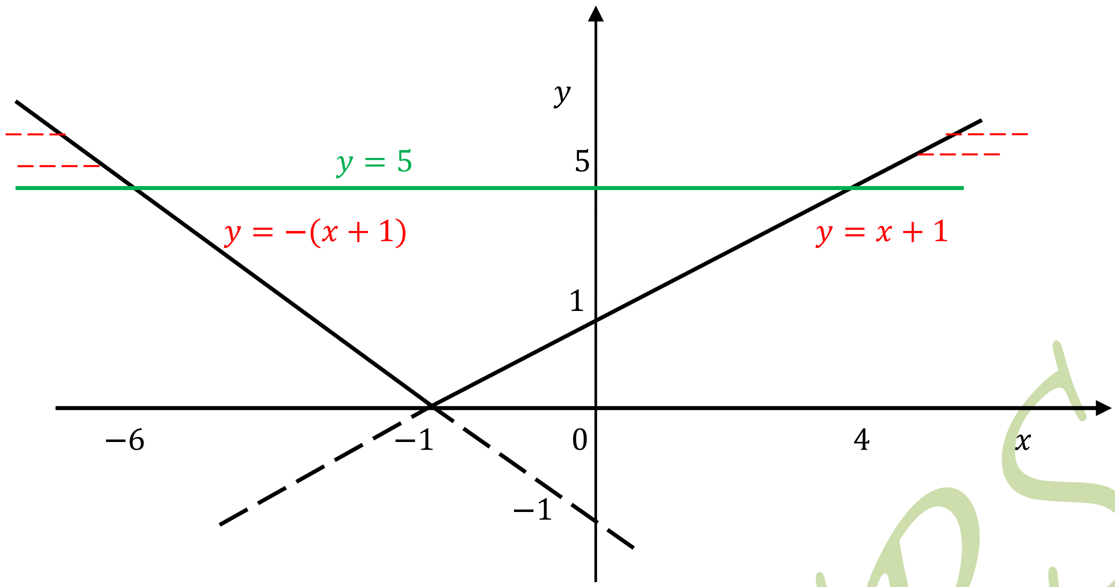
$|x+1| \geq 5$

LHS

$y = x + 1$: When $x = 0$; $y = 1$ and when $y = 0$; $x = -1$

RHS

$y = 5$.



From the diagram above we calculate the critical values as follows:

$-(x+1)=5$ and $x+1=5$

$-x-1=5$ $\quad x=5-1$

$x=-6$ $\quad x=4$

Now to solve $|x+1| \geq 5$ we consider the shaded part.

$\therefore$The solution set is $\{x \in \Re: x \leq -6 \;\text{ or }\; x \geq 4\}$

Follow Up Exercise

Question 1

Solve the inequality $|x+1| \leq 2$

Answer: $\{x \in \Re: -3 \leq x \leq 1\}$

Question 2

Solve the inequality $|3x-7| < 5$

Answer: $\{x \in \Re: 2/3 < x < 4\}$

Question 3

Solve the inequality $|x - 5| > 1$

Answer: $\{x \in \Re: x < 4 \text{ or } x > 6\}$

Question 4

Solve the inequality $|3x| \geq 2$

Answer: $\{x \in \Re: x \leq -2/3 \text{ or } x \geq 2/3\}$

Question 5

Solve the inequality $|6x - 1| \leq 11$

Answer: $\{x \in \Re: -5/2 \leq x \leq 2\}$

Question 6

Solve the inequality $|x + 8| < 10$

Answer: $\{x \in \Re: -18 < x < 2\}$

Question 7

Solve the inequality $|2x - 4| \geq 2$

Answer: $\{x \in \Re: x \leq 1 \text{ or } x \geq 3\}$

Question 8

Solve the inequality $|3^x - 8| < 0.5$

Answer: $\{x \in \Re: 1.83 < x < 1.95\}$

Question 9

Solve the inequality $|8 - 3x| < 2$

Answer: $\{x \in \Re: 2 < x < 10/3\}$

Question 10

Solve the inequality $|9 - 2x| < 1$

Answer: $\{x \in \Re: 4 < x < 5\}$

Question 11

Solve the inequality $|2x - 7| > 3$

Answer: $\{x \in \Re: x < 2 \text{ or } x > 5\}$

Question 12

Solve the inequality $|2^x - 8| < 5$

Answer: $\{x \in \Re: 1.58 < x < 3.70\}$

Question 13

Given that the solution of $|x - 2a| < b$ is $-3 < x < 11$, find the values of a and b.

Answer: $a = 2$ and $b = 7$

Question 14

Solve the inequality $|3^{-x} - 54| < 27$

Answer: $\{x \in \Re: -4 < x < -3\}$

Question 15

Solve the inequality $|2^x - 20| \leq 12$

Answer: $\{x \in \Re: 3 \leq x \leq 5\}$

Question 16

Solve the inequality $|3^x - 54| \geq 27$

Answer: $\{x \in \mathfrak{R}:\ x \leq 3 \text{ or } x \geq 4\}$

Question 17

Solve the inequality $|2^{-x} - 20| > 12$

Answer: $\{x \in \mathfrak{R}: x < -5 \text{ or } x > -3\}$

Case 3 – Modulus on one side

$|x \pm a| < bx \pm c$ or $|x \pm a| > bx \pm c$

Graphical method is advisable to use when the modulus sign is on one side of the inequality.

NOTE

(a) For $|x \pm a| < bx + c$

Squaring comes with a condition that RHS should be greater than zero since the LHS is already positive

Hence RHS:

$$bx \pm c > 0.$$

Therefore the required solution will be all values of $x > \pm\frac{c}{b}$.

(b) For $|x \pm a| > bx + c$

Squaring comes with a condition that RHS should be less than zero since the LHS is already positive

Hence RHS:

$$bx \pm c < 0.$$

Therefore the required solution will be all values of $x < \pm\frac{c}{b}$.

Method 1 - Best Method

Graphical

Method 2

Squaring both sides considering the fact that $x > \pm\frac{c}{b}$.

Solved Problems

Question 1

Solve the inequality $|x-2| < 3x+2$

Suggested Solution

Method 1

Graphical

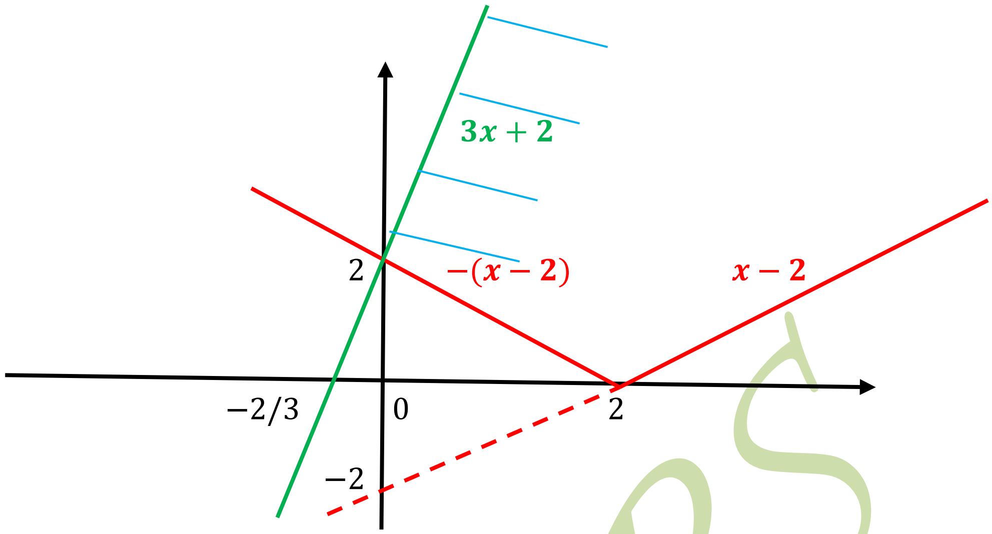
Drawing the graphs of $y = |x-2|$ and $y = 3x+2$.

Considering the function $y = x-2$:

When $x = 0$; $y = -2$ and when $y = 0$, $x = 2$.

Considering the function $y = 3x+2$:

When $x = 0$; $y = 2$ and when $y = 0$, $x = -\frac{2}{3}$.



Finding the critical value from the graphs:

We equate the equations of the intersecting graphs.

$3x + 2 = -(x - 2)$

$3x + 2 = -x + 2$

$4x = 0$

$\therefore x = 0$

Now from the graphs we see that the shaded region satisfies $|x - 2| < 3x + 2$:

$\therefore$ The solution set is $\{x \in \Re : x > 0\}$

Method 2

Squaring both sides

CONDITION

In order to use squaring method, we must take note of the condition which follows:

$3x + 2 > 0$

$3x > -2$

$x > -\frac{2}{3}$.

Therefore all the solutions of $x > -\frac{2}{3}$ are valid.

Now:

$(x-2)^2 < (3x+2)^2$

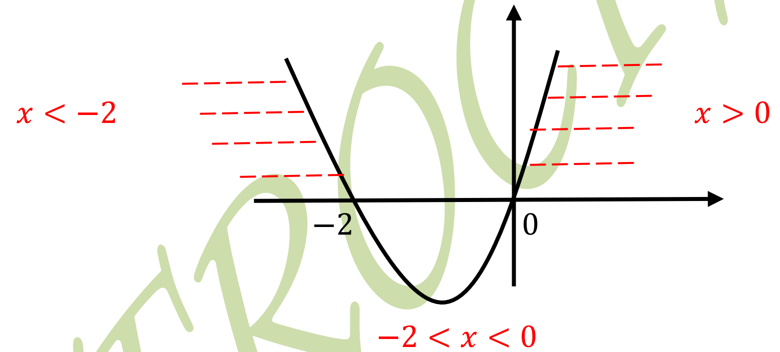
$x^2 - 4x + 4 < 9x^2 + 12x + 4$

$0 < 9x^2 - x^2 + 12x + 4x + 4 - 4$

$8x^2 + 16x > 0$

$8x(x+2) > 0$

The critical values are $x = 0$ and $x = -2$



Since the inequality sign is > 0 which means that the required solutions are those above the $x - axis$.

$\Rightarrow x < -2$ or $x > 0$ BUT

Taking note of the condition above, only all the solutions of $x > -\frac{2}{3}$ are required.

$\therefore$ The solution set is $\{x \in \Re:\ x > 0\}$

Question 2

Sketch, on a single diagram, the graphs of $x + 2y = 6$ and $y = |x + 2|$.

Hence solve the inequality $|x + 2| < \frac{1}{2}(6 - x)$.

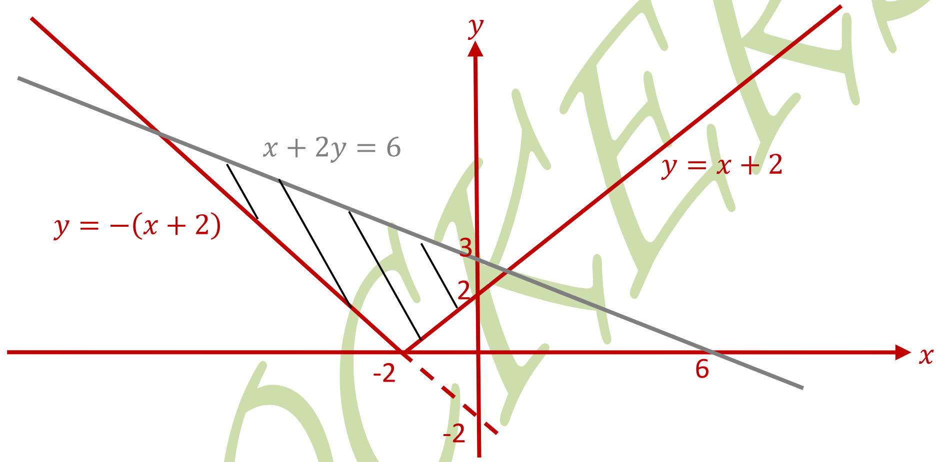
Suggested Solution

$x + 2y = 6$

When $x = 0; y = 3$ and when $y = 0; x = 6$

$y = x + 2$

When $x = 0; y = 2$ and when $y = 0; x = -2$



Critical Values:

$-(x + 2) = \frac{1}{2}(6 - x)$

$-2x - 4 = 6 - x$

$-2x + x = 6 + 4$

$-x = 10$

$x = -10$

$(x + 2) = \frac{1}{2}(6 - x)$

$2x + 4 = 6 - x$

$2x + x = 6 - 4$

$3x = 2$

$x = \frac{2}{3}$

From the graph the solution is the shaded region i.e.

$$x \in \mathbb{R}: -10 < x < \frac{2}{3}$$

Question 3

Solve the inequality $|x-2| < 2x-3$

Suggested Solution

Method 1

Graphical

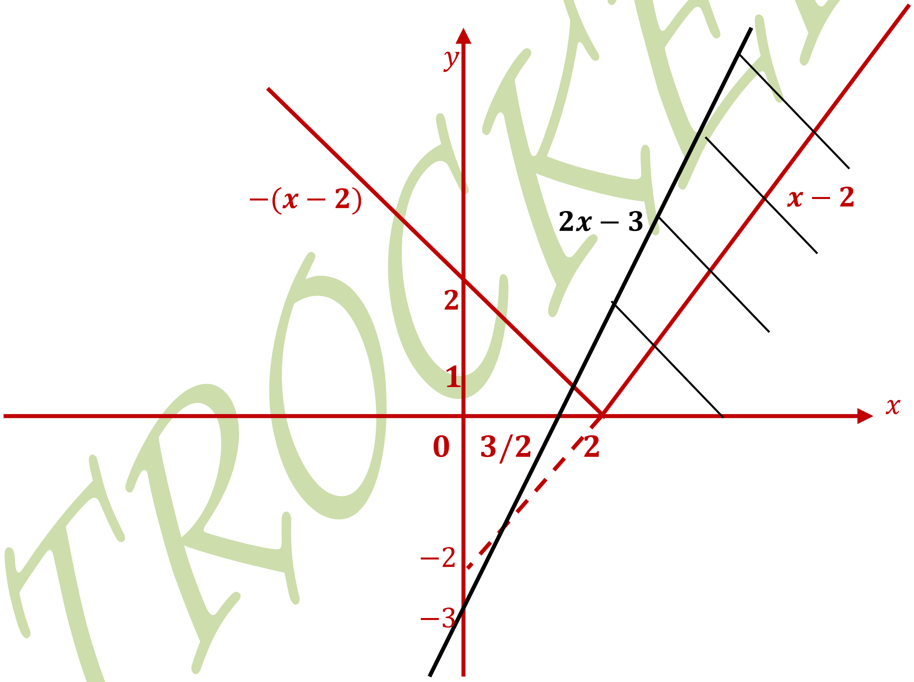
Drawing the graphs of $y = |x-2|$ and $y = 2x-3$.

Considering the function $y = x-2$:

When $x = 0$; $y = -2$ and when $y = 0$, $x = 2$.

Considering the function $y = 2x-3$:

When $x = 0$; $y = -3$ and when $y = 0$, $x = \frac{3}{2}$.



Finding the critical value(s) from the graphs:

We equate the equations of the intersecting graphs.

$2x-3 = -(x-2)$

$2x-3 = -x+2$

$3x = 5$

$\therefore x = \frac{5}{3}$

Now from the graphs we see that the shaded region satisfies $|x-2| < 2x-3$

$\therefore$ The solution set is $\left\{x \in \mathbb{R}: x > \frac{5}{3}\right\}$

Method 2

$|x-2| < 2x-3$

Squaring both sides

CONDITION

In order to use squaring method, we must take note of the condition which follows:

$2x-3 > 0$

$x > \frac{3}{2}$

Therefore all the solutions of $x > \frac{3}{2}$ are valid.

Now:

$(x-2)^2 < (2x-3)^2$

$x^2 - 4x + 4 < 4x^2 - 12x + 9$

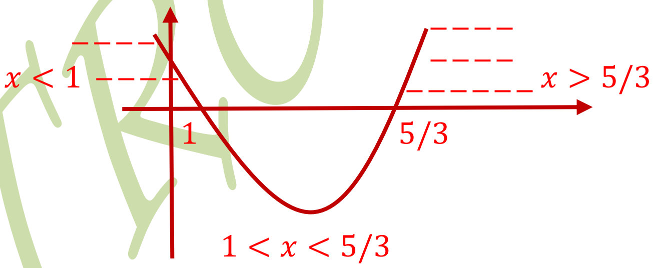
$0 < 4x^2 - x^2 - 12x + 4x + 9 - 4$

$3x^2 - 8x + 5 > 0$

$3x^2 - 3x - 5x + 5 > 0$

$(x-1)(3x-5) > 0$

The critical values are $x = 1$ and $x = \frac{5}{3}$



Since the inequality sign is > 0 which means that the required solutions are those above the x-$axis$.

$\Rightarrow x < 1$ and $x > \frac{5}{3}$

Taking note of the condition above, only all the solutions of $x > \frac{3}{2}$ are required.

$\therefore$ The solution set is $\left\{x \in \mathbb{R}: x > \frac{5}{3}\right\}$

Question 4

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Solve the inequality $|2x + 2| > 1 - 4x$

Suggested Solution

Method 1

Graphical

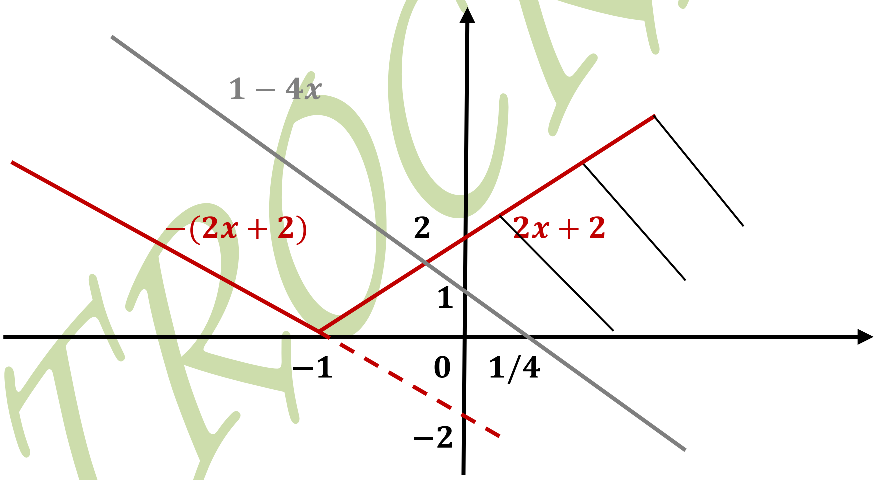
Drawing the graphs of $y = |2x + 2|$ and $y = 1 - 4x$.

Considering the function $y = 2x + 2$:

When $x = 0$; $y = 2$ and when $y = 0$, $x = -1$.

Considering the function $y = 1 - 4x$:

When $x = 0$; $y = 1$ and when $y = 0$, $x = \frac{1}{4}$.



Finding the critical value(s) from the graphs:

We equate the equations of the intersecting graphs.

$2x + 2 = 1 - 4x$

$2x + 4x = 1 - 2$

$6x = -1$

$\therefore x = -\frac{1}{6}$

Now from the graphs we see that the shaded region satisfies $|2x + 2| > 1 - 4x$

$\therefore$ The solution set is $\left\{x \in \mathbb{R}: x > -\frac{1}{6}\right\}$

Method 2

Squaring both sides

CONDITION

In order to use squaring method, we must take note of the condition which follows:

$1 - 4x < 0$

$1 < 4x$

$x > \frac{1}{4}$

Therefore all the solutions of $x > \frac{1}{4}$ are valid.

Now:

$(2x + 2)^2 > (1 - 4x)^2$

$4x^2 + 8x + 4 > 1 - 8x + 16x^2$

$0 > 16x^2 - 4x^2 - 8x - 8x + 1 - 4$

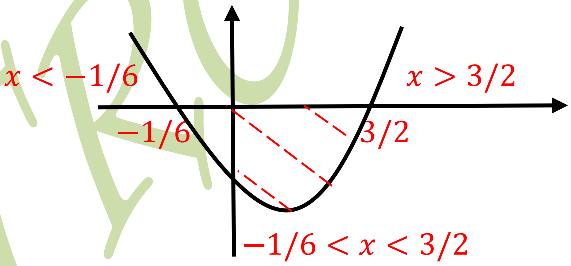
$12x^2 - 16x - 3 < 0$

$12x^2 - 18x + 2x - 3 < 0$

$6x(2x - 3) + 1(2x - 3) < 0$

$(2x - 3)(6x + 1) < 0$

The critical values are $x = -\frac{1}{6}$ and $x = \frac{3}{2}$



Since the inequality sign is < 0 which means that the required solutions are those below the $x - axis$.

$\Rightarrow -\frac{1}{6} < x < \frac{3}{2}$ NB: $x > -\frac{1}{6}$ and $x < \frac{3}{2}$

Taking note of the condition above, only all the solutions of $x > \frac{1}{4}$ are required.

$\therefore$ The solution set is $\left\{x \in \mathbb{R}: x > -\frac{1}{6}\right\}$

Follow Up Exercise

Question 1

Solve the inequality $|x-2| > 3-2x$

Answer: $\{x \in \Re:\ x < 1\}$

Question 2

Solve the inequality $2x \leq |x-1|$

Answer: $\{x \in \Re:\ x \geq 1/3\}$

Question 3

Solve the inequality $|2-4x| > -x$

Answer: $\{x \in \Re:\ x > 0\}$

Question 4

Solve the inequality $x+1 < |x-4|$

Answer: $\{x \in \Re:\ x < 3/2\}$

Question 5

Solve the inequality $x-5 \leq |3-x|$

Answer: $\{x \in \Re:\ x \geq 0\}$

Question 6

Solve the inequality $|x-2| < 3x-3$

Answer: $\{x \in \Re:\ x < 5/4\}$

Question 7

Solve the inequality $|x+6| < 3x+2$

Answer: $\{x \in \Re: x > 2\}$

Question 8

Solve the inequality $x+6 > |3x+2|$

Answer: $\{x \in \Re: -2 < x < 2\}$

Question 9

Solve the inequality $|2x + 1| \geq 3x + 9$

Answer: $\{x \in \Re: x \geq -2\}$

Question 10

Solve the inequality $|x + 1| \leq -2x - 5$

Answer: $\{x \in \Re: x \leq -4\}$

Question 11

Solve the inequality $|x + 5| > 1 - x$

Answer: $\{x \in \Re: x < -2\}$

Question 12

ZIMSEC JUNE 2019 PAPER 1

Solve the inequality $|x - 1| < 2x - 4$

Answer: $\{x \in \Re: x > 3\}$

Case 4 – Modulus on both sides

$|x \pm a| < |bx \pm c|$ or $|x \pm a| > |bx \pm c|$

- When the modulus sign is on both sides of the inequality we use either **graphical** method or **squaring** method

Solved Problem

Example

Solve the inequality $|2x + 1| < |x - 3|$.

Suggested Solution

Method 1

Squaring both side

$|2x+1| < |x-3|$

$(2x+1)^2 < (x-3)^2$

$4x^2 + 4x + 1 < x^2 - 6x + 9$

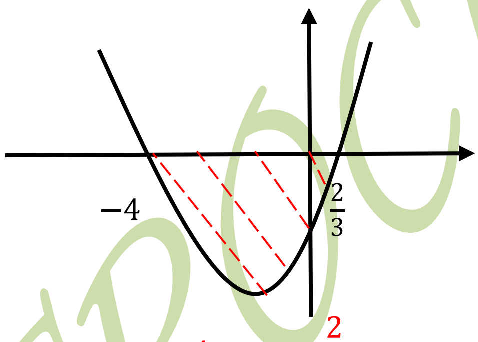
$3x^2 + 10x - 8 < 0$

$3x^2 + 12x - 2x - 8 < 0$

$3x(x+4) - 2(x+4) < 0$

$(3x-2)(x+4) < 0$

The critical values are $x = \frac{2}{3}$ and $x = -4$



$$-4 < x < \frac{2}{3}$$

Since the inequality sign is < 0 which means that the required solutions are those below the $x - axis$.

$\therefore$ The solution set is $\left\{x \in \Re: -4 < x < \frac{2}{3}\right\}$

Method 2

Graphical

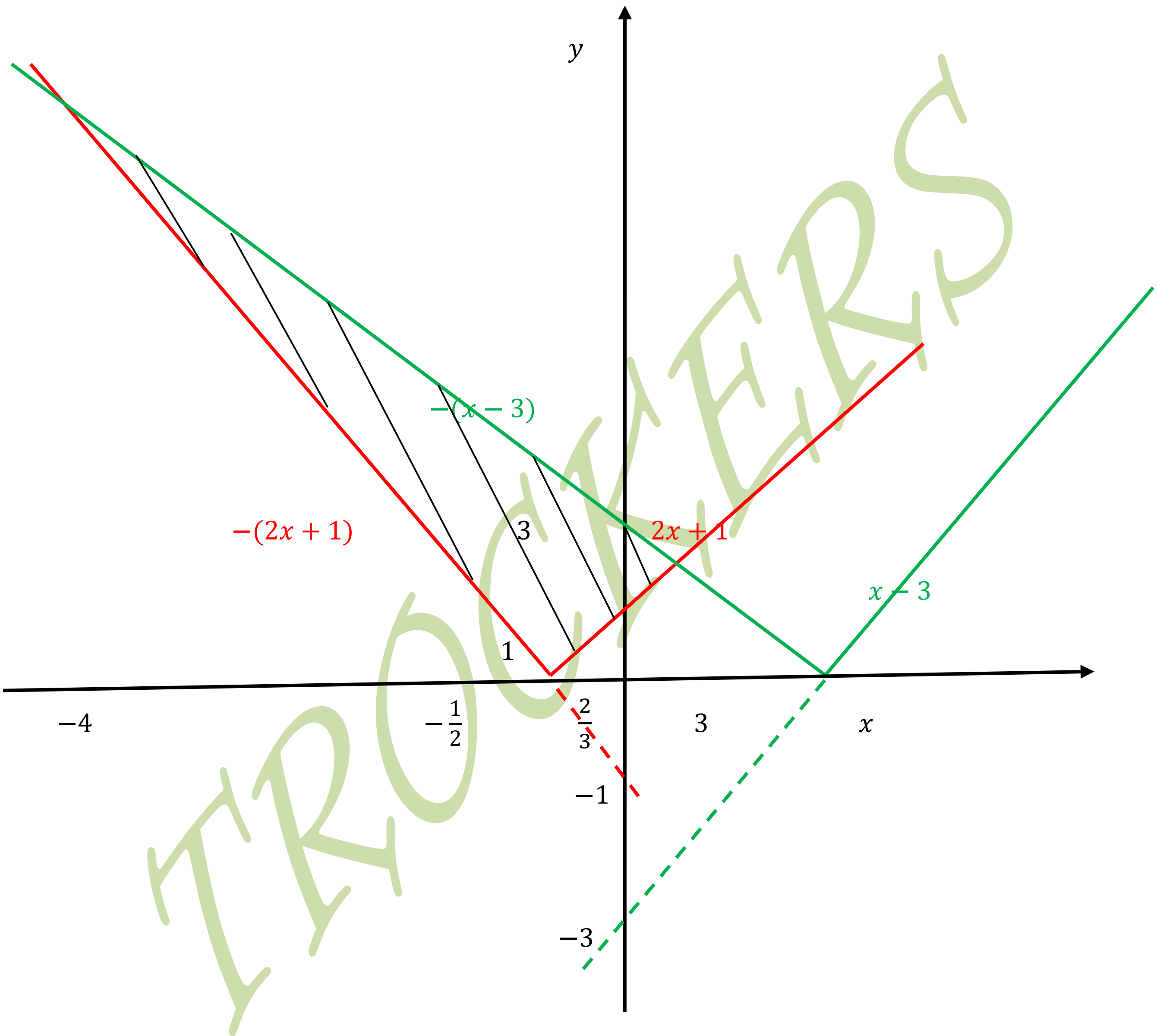
Drawing the graphs of $y = |2x+1|$ and $y = |x-3|$.

Considering the function $y = 2x + 1$:

When $x = 0$; $y = 1$ and when $y = 0$, $x = -\frac{1}{2}$.

Considering the function $y = x - 3$:

When $x = 0$; $y = -3$ and when $y = 0$, $x = 3$



From the diagram above we calculate the critical values as follows:

$-(2x+1) = -x+3$	and	$2x+1 = -x+3$
$-2x-1 = -x+3$		$2x+x = 3-1$
$x = -4$		$x = \frac{2}{3}$

Now to solve $|2x + 1| < |x - 3|$ we consider the shaded part.

$\therefore$ The solution set is $\left\{x \in \Re: -4 < x < \frac{2}{3}\right\}$

Follow Up Exercise

Question 1

Solve the inequality $|3x - 1| \geq |2x + 5|$

Answer: $\{x \in \Re: -4/5 < x < 6\}$

Question 2

Solve the inequality $|2x - 5| < 3|2x + 1|$

Answer: $\{x \in \Re: -2 < x < 5/3\}$

Question 3

Solve the inequality $2|x - 2| \geq |3x + 1|$

Answer: $\{x \in \Re: -5 \leq x \leq 3/5\}$

Question 4

Solve the inequality $|2x - 5| < |x + 3|$

Answer: $\{x \in \Re: 2/3 < x < 8\}$

Question 5

Solve the inequality $|x - 4| < 2|3x + 1|$

Answer: $\{x \in \Re: x < -6/5 \text{ or } x > 2/7\}$

Question 6

Solve the inequality $|2x - 1| \leq |x|$

Answer: $\{x \in \Re: x \leq 1/3 \text{ or } x \geq 1\}$

Question 7

Solve the inequality $|x - 2| > 3|2x + 1|$

Answer: $\{x \in \Re: -1 < x < -1/7\}$

Question 8

Solve the inequality $|x - 3| > |2x|$

Answer: $\{x \in \Re: -3 < x < 1\}$

Question 9

Solve the inequality $|x + 2| < |5 - 2x|$

Answer: $\{x \in \Re: x < 1 \text{ or } x > 7\}$

Question 10

Solve the inequality $|x| \geq |5 + 2x|$

Answer: $\{x \in \Re: x < -5 \text{ or } x > -5/3\}$

Question 11

Solve the inequality $|3x - 2| < |x + 5|$

Answer: $\{x \in \Re: -3/4 < x < 7/2\}$

Question 12

Solve the inequality $|x + 1| \geq |x|$

Answer: $\{x \in \Re: x > -1/2\}$

Question 13

Solve the inequality $|2x + 1| < |x|$

Answer: $\{x \in \Re: -1 < x < 7 - 1/3\}$

Question 14

Solve the inequality $|x + 1| \geq |x|$

Answer: $\{x \in \Re: x \geq -1/2\}$

Question 15

Solve the inequality $|x| > |3x - 2|$

Answer: $\{x \in \Re:\ 1/2 < x < 1\}$

Question 16

Solve the inequality $|x - 3a| > |x - a|$ given that a is a positive constant

Answer: $\{x \in \Re:\ x < 2a\}$

Question 17

It is given that the variable x is such that $1.3^{2x} < 80$ and $|3x - 1| > |3x - 10|$. Find the set of possible value of x, giving your answer in the form $a < x < b$, where a and b are correct to 3 significant figures.

Answer: $\{x \in \Re:\ 1.83 < x < 8.35\}$

ASANTE SANA

*******THERE IS A LIGHT AT THE END OF EVERY TUNNEL *******

CONSTRUCTIVE COMMENTS ON THE FORM OF THE PRESENTATION, INCLUDING ANY OMISSIONS OR ERRORS, ARE WELCOME.

ENJOY

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