Linear Law

Compiled by: Nyasha P. Tarakino (Trockers)

+263772978155/+263717267175

ntarakino@gmail.com

13 MARCH 2020

SYLLABUS (6042) REQUIREMENTS

- reduce an equation to appropriate linear form in solving problems (such as $y = ax^2 + b$ when y is plotted against x^2)

LINEAR LAW

Definition: It is a process of converting a set of two variables which are related nonlinearly to a linear equation of the form $Y = mx + c$.

Modification/Transformation of Laws

All the non-linear form laws with variables x and y must be changed, transformed or modified to a linear form $Y = mx + c$ where:

- m is the gradient and
- $+c$ is the vertical/ y-axis intercept (where the graph cuts or crosses the y- axis).

Illustration

If it is necessary to plot graphs of the given laws in a modified form in order to verify them, state

(i) what should be plotted on the vertical axis,

(ii) what should be plotted on the horizontal axis,

(iii)the gradient and

(iv)the vertical axis intercept.

(a) $y = ax^2 + b$

$y = ax^2 + b$ in comparison to $Y = mx + c$ yields the following results:

(i) Vertical axis label: y

(ii) Horizontal axis label: x^2

(iii)Gradient: a

(iv)Vertical axis intercept: $+b$

(b) $y = ax^2 + bx$

$y = ax^2 + bx$

$\frac{y}{x} = \frac{ax^2}{x} + \frac{bx}{x}$

$\frac{y}{x} = ax + b$ in comparison to $Y = mx + c$ yields the following results:

(i) Vertical axis label: $\frac{y}{x}$

(ii) Horizontal axis label: x

(iii)Gradient: a

(iv)Vertical axis intercept:$+b$

(c) $y = a\sqrt{x} + b$

$y = a\sqrt{x} + b$ in comparison to $Y = mx + c$ yields the following results:

(i) Vertical axis label: y

(ii) Horizontal axis label: $\sqrt{x}$

(iii)Gradient: a

(iv)Vertical axis intercept: $+b$

(d) $y = \frac{a}{x} + b$

$y = \frac{a}{x} + b$

$y = a\left(\frac{1}{x}\right) + b$ in comparison to $Y = mx + c$ yields the following results:

(i) Vertical axis label: y

(ii) Horizontal axis label: $\frac{1}{x}$

(iii)Gradient: a

(iv)Vertical axis intercept:$+b$

(e) $y = \frac{a}{x} + bx$

$y = \frac{a}{x} + bx$

$\frac{y}{x} = \frac{a}{x^2} + b$

$\frac{y}{x} = a\left(\frac{1}{x^2}\right) + b$ in comparison to $Y = mx + c$ yields the following results:

(i) Vertical axis label: $\frac{y}{x}$

(ii) Horizontal axis label: $\frac{1}{x^2}$

(iii)Gradient: a

(iv)Vertical axis intercept:$+b$

(f) $y = \frac{a}{x^2} + b$

$y = \frac{a}{x^2} + b$

$y = a\left(\frac{1}{x^2}\right) + b$ in comparison to $Y = mx + c$ yields the following results:

(i) Vertical axis label: y

(ii) Horizontal axis label: $\frac{1}{x^2}$

(iii)Gradient: a

(iv)Vertical axis intercept:$+b$

(g) $y = \frac{x}{ax+b}$

$y = \frac{x}{ax + b}$

$y(ax + b) = x$

$axy + by = x$

$y + \frac{b}{a}\left(\frac{y}{x}\right) = \frac{1}{a}$

$y = -\frac{b}{a}\left(\frac{y}{x}\right) + \frac{1}{a}$ in comparison to $Y = mx + c$ yields the following results:

(i) Vertical axis label: y

(ii) Horizontal axis label: $\frac{y}{x}$

(iii)Gradient: $-\frac{b}{a}$

(iv)Vertical axis intercept: $+\frac{1}{a}$

(h) $y = ax^b$

$y = ax^b$

$logy = log(ax^b)$

$logy = log(a) + log(x^b)$

$logy = blogx + loga$ in comparison to $Y = mx + c$ yields the following results:

NB: Depending on the question; in other cases you may be instructed to use natural logarithms i.e. $Iny = In(ax^b)$

(i) Vertical axis label: $logy$

(ii) Horizontal axis label: $logx$

(iii)Gradient: b

(iv)Vertical axis intercept:$+loga$

(i) $y = ab^x$

$y = ab^x$

$logy = log(ab^x)$

$logy = log(a) + log(b^x)$

$logy = xlogb + loga$ in comparison to $Y = mx + c$ yields the following results:

NB: Depending on the question; in other cases you may be instructed to use natural logarithms i.e. $Iny = In(ab^x)$

(i) Vertical axis label: $logy$

(ii) Horizontal axis label: x

(iii)Gradient: $logb$

(iv)Vertical axis intercept: $+loga$

(j) $y = ae^{bx}$

$y = ae^{bx}$

$Iny = In(ae^{bx})$

$Iny = Ina + Ine^{bx}$

$Iny = Ina + bx$

$Iny = bx + Ina$ in comparison to $Y = mx + c$ yields the following results:

(i) Vertical axis label: Iny

(ii) Horizontal axis label: x

(iii)Gradient: b

(iv)Vertical axis intercept. $+Ina$

Straight Line

After transforming the given law, at times it is necessary to draw the straight line graph in order to verify the linear relationship.

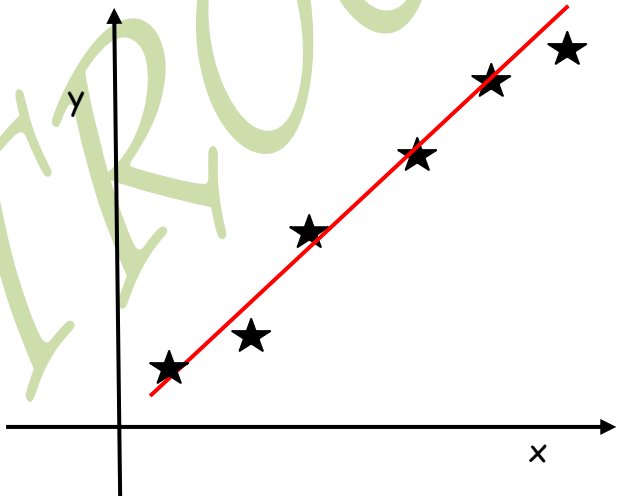
Steps to draw a straight line

- Use a graph paper
- Construct a table of values (using the new law)
- Choose a suitable scale, unless it is already specified. Label both axes
- Choose the suitable line of best fit
- Determine the gradient – m and y-intercept c.

The concept of line of best fit

Straight line must pass through many points where possible. If points are plotted and they do not lie on the straight line, the line should be drawn is such a way that points are more or less the same on both sides.

Illustration



Worked Examples

Question 1

ZIMSEC JUNE 2000 PAPER

Two variable quantities x and y are related by the equation $y = a(b^x)$, where a and b are constants. When a graph is plotted showing values of Iny on the vertical axis and values of x on the horizontal axis, the points lie on a straight line having gradient 1.8 and crossing the vertical axis at the point $(0, 4.1)$. Find the values of a and b. **[5]**

Suggested Solution

$y = ab^x$

$Iny = In(ab^x)$

$Iny = In(a) + In(b^x)$

$Iny = Ina + xInb$

$\Rightarrow Gradient = Inb$ and $Vertical\ axis\ intercept = Ina$

Now:

$Ina = 4.1$

$\Rightarrow a = e^{4.1}$

$a = 60.3402876$

$\therefore a = 60$ (to 2 $s.f.$)

Also:

$Inb = 1.8$

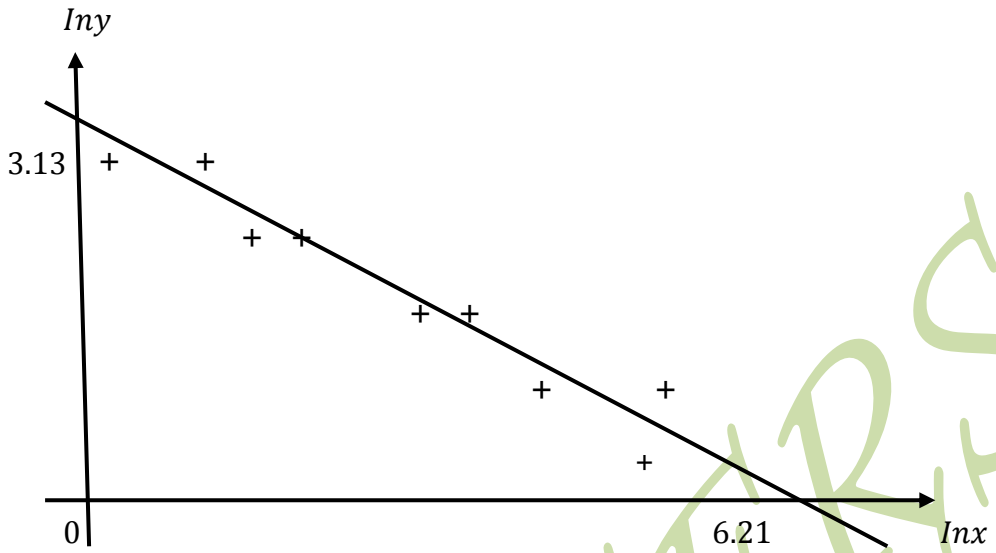
$\Rightarrow b = e^{1.8}$

$b = 6.049647464$

$\therefore b = 6.0$ (to 2 $s.f.$)

Question 2

ZIMSEC NOVEMBER 1996 PAPER 1



Values of the related variables x and y were measured in an experiment, and the results were plotted on a graph of Iny against Inx. The points were found to lie approximately on a straight line crossing the axes at $Iny = 3.13$ and $Inx = 6.21$, as shown in the diagram. Show that this linear model implies a relation of the form $y = Ax^n$, and find the values of A and n.

Suggested Solution

$$Gradient = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3.13 - 0}{0 - 6.21}$$

$$= -\frac{3.13}{6.21}$$

$$= -\frac{313}{621}$$

Now let the general point on the line be $(Inx; Iny)$

Now choose either $(6.21;\ 0)$ or $(0;\ 3.13)$ and calculate the gradient

$$\Rightarrow Gradient = \frac{Iny - y_1}{Inx - x_1}$$

$$\Rightarrow \frac{Iny - 3.13}{Inx - 0} = -\frac{313}{621}$$

$$\Rightarrow \frac{Iny - 3.13}{Inx} = -\frac{313}{621}$$

$$\Rightarrow Iny - 3.13 = -\frac{313}{621}Inx$$

$$\Rightarrow Iny = -\frac{313}{621}Inx + 3.13$$

$$\Rightarrow e^{Iny} = e^{\left(-\frac{313}{621}Inx + 3.13\right)}$$

$$\Rightarrow y = e^{\left(Inx^{-\frac{313}{621}} + 3.13\right)}$$

$$\Rightarrow y = e^{\left(Inx^{-\frac{313}{621}}\right)} \times e^{(3.13)}$$

$$\Rightarrow y = x^{-\frac{313}{621}} \times e^{(3.13)}$$

$$\Rightarrow y = e^{(3.13)} \times x^{-\frac{313}{621}}$$

$$\Rightarrow y = 22.87397954x^{-0.504025764}$$

$$\Rightarrow y = 23x^{-0.50} \text{ (as required)}$$

$$\therefore A = 23 \text{ (to } 2s.f.) \text{ and } n = -0.50 \text{ (to } 2\ s.f.)$$

Question 3

ZIMSEC NOVEMBER 2014 PAPER 1

The table below shows the values of a variable p obtained experimentally from the values of q.

q	2	3	4	5
p	3.55	1.93	1.26	0.90

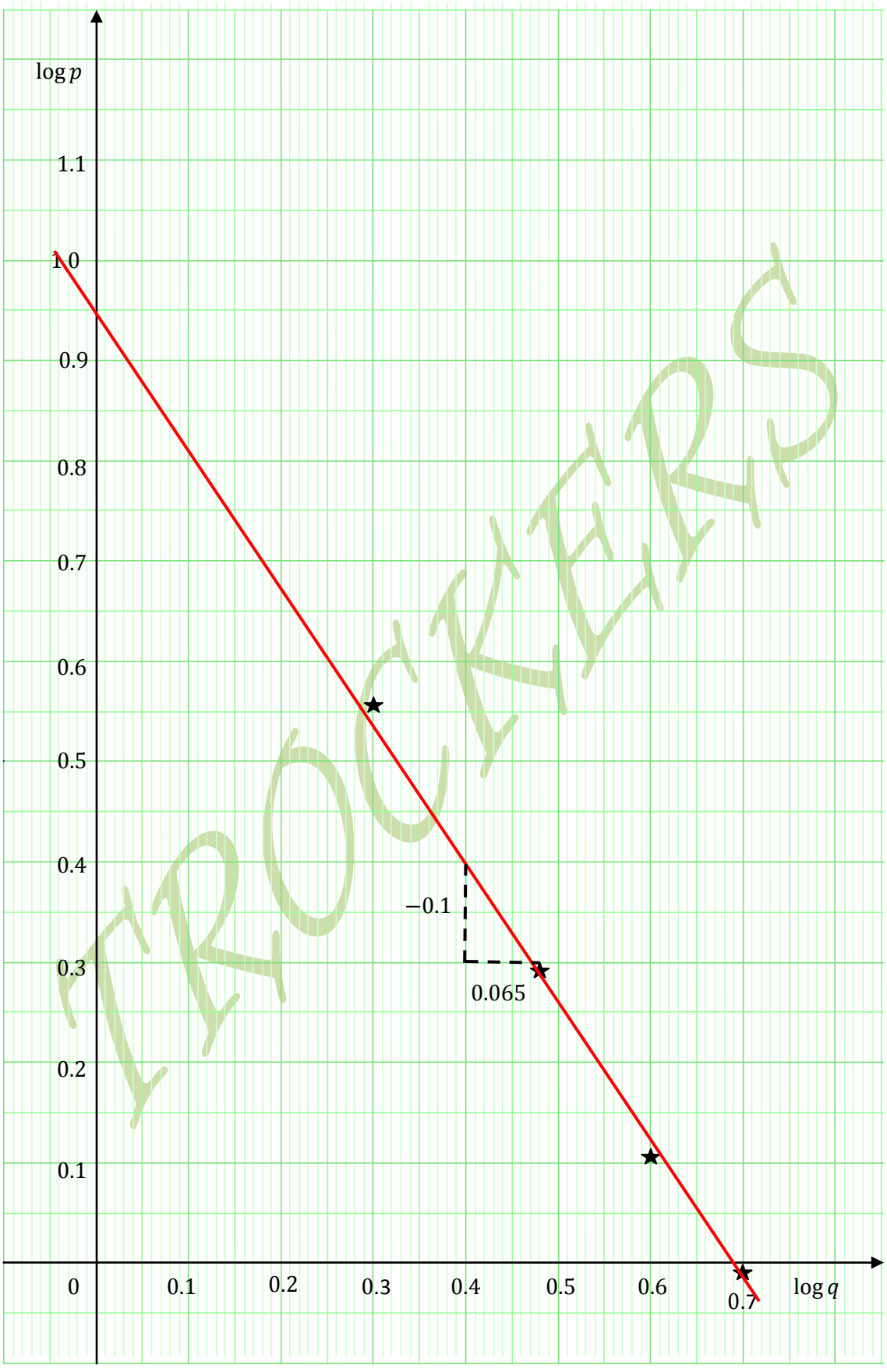
The variables p and q are related by the relation $logp = blogq + loga$.

By plotting the graph of $\log p$ against $\log q$, determine the approximate values of a and b. **[6]**

Suggested Solution

$logp = blogq + loga$

$logq$	0.30	0.48	0.60	0.70
$logp$	0.55	0.29	0.10	-0.05


log p
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
log q
−0.1
0.065

$logp = blogq + loga$

$\Rightarrow Gradient = b$ and $Vertical\ axis\ intercept = loga$

Now:

$loga = 0.95$

$\Rightarrow a = 10^{0.95}$

$\Rightarrow a = 9.925093813$

$\therefore a = 9.9$ (to 2 $s.f.$)

Also:

$Gradient = \frac{\Delta y}{\Delta x}$ (check on the graph – dotted triangle)

$= \frac{-0.1}{0.065}$

$= -1.538461538$

$\therefore b = -1.5$ (to 2 $d.p.$)

Question 4

ZIMSEC NOVEMBER 2015 PAPER 1

The variables x and y satisfy a relationship of the form $y = ax^2 + bx$, where a and b are constants.

The values obtained in an experiment are shown in the table

x	1	2	3	4	5
y	1.0	8.4	22.3	39.1	65.8

Plot the graph of $\frac{y}{x}$ against x and determine the values of a and b. **[5]**

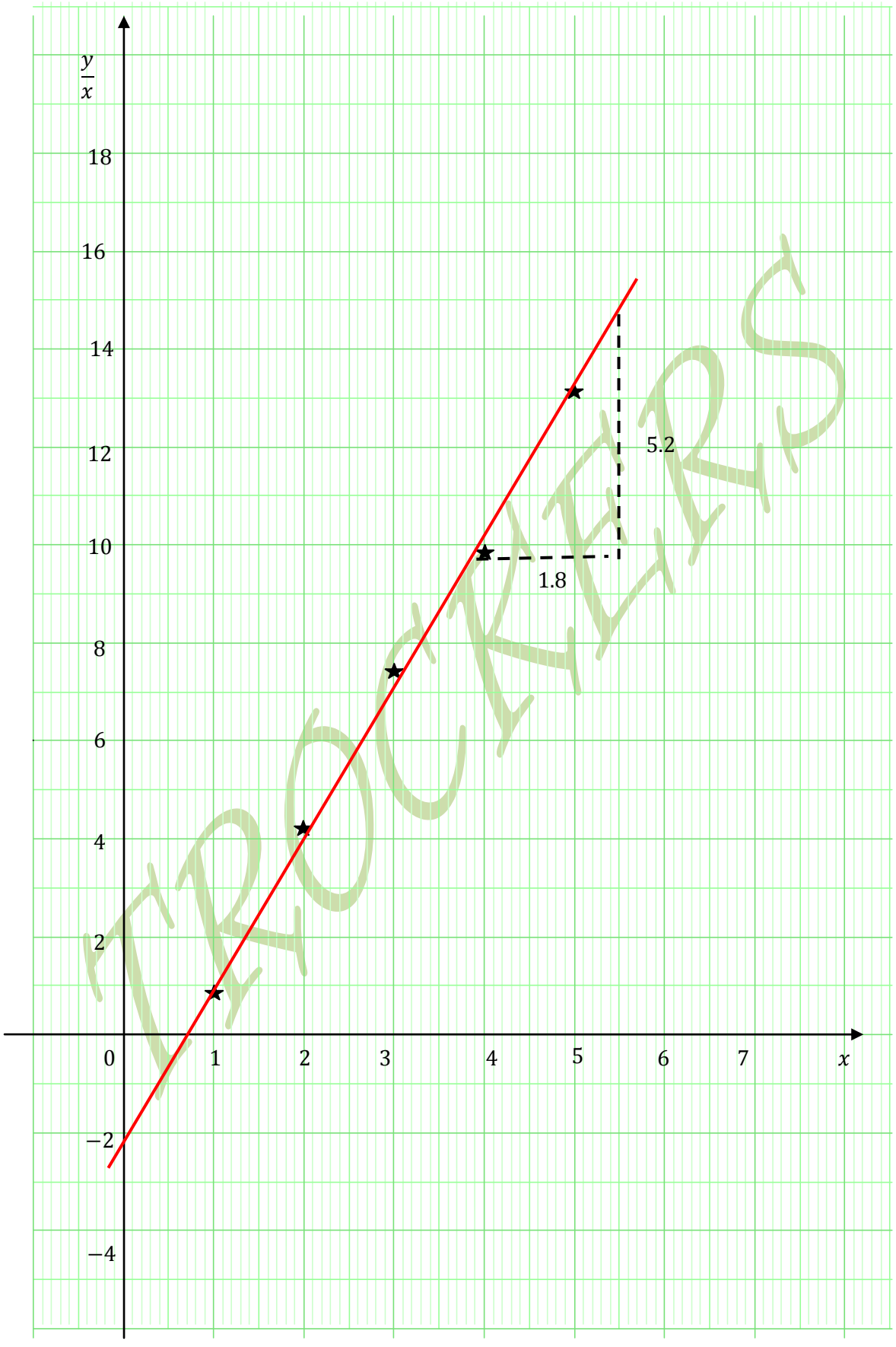
Suggested Solution

$y = ax^2 + bx$

$\frac{y}{x} = \frac{ax^2}{x} + \frac{bx}{x}$

$\frac{y}{x} = ax + b$

x	1	2	3	4	5
$\frac{y}{x}$	1.0	4.2	7.4	9.8	13.1



$\frac{y}{x} = ax + b$

$\Rightarrow Gradient = a$ and $Vertical\ axis\ intercept = b$

Now:

y-intercept $= -2$

$\therefore b = -2$

Also:

$Gradient = \frac{\Delta y}{\Delta x}$ (check on the graph – dotted triangle)

$= \frac{5.2}{1.8}$

$= 2.88888889$

$\therefore a = 2.9$ (to 2 $d.p.$)

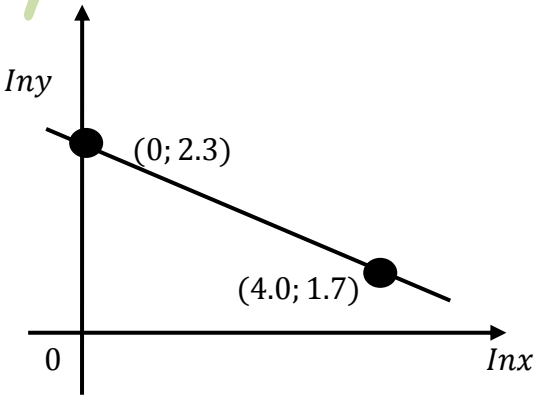
Question 5

CAMBRIDGE NOVEMBER 2002 PAPER 2

The variables x and y are related by the equation

$$y = Ax^n,$$

where A and n are constants.



When a graph is plotted values of Iny on the vertical axis and values of Inx on the horizontal axis, the points lie on a straight line. This line crosses the vertical axis at the point $(0;\ 2.3)$ and also passes through the point $(4.0;\ 1.7)$ as shown in the diagram. Find the values of A and n. **[5]**

Suggested Solution

$y = Ax^n$

$Iny = In(Ax^n)$

$Iny = InA + In(x^n)$

$Iny = nInx + InA$

$\Rightarrow Gradient = n$ and $Vertical\ axis\ intercept = InA$

Now:

$InA = 2.3$

$\Rightarrow A = e^{2.3}$

$A = 9.974182455$

$\therefore A = 10.0$ (to 2 $s.f.$)

Also:

$$Gradient = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2.3 - 1.7}{0 - 4.0}$$

$$= -\frac{0.6}{4.0}$$

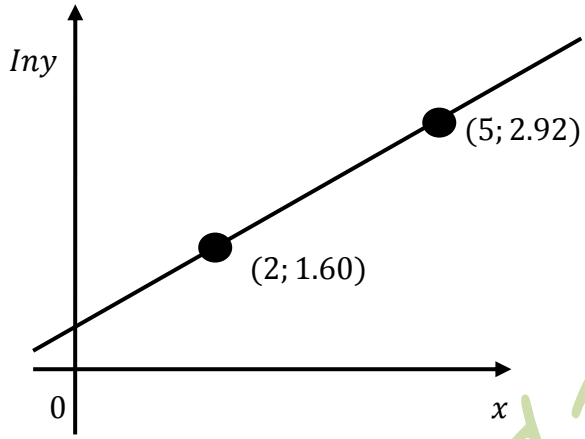
$$= -\frac{3}{20}$$

$\Rightarrow n = -0.15$

$\therefore n = -0.15$ (to 2 $s.f.$)

Question 6

CAMBRIDGE JUNE 2015 PAPER 2



The variables x and y satisfy the equation

$$y = Ae^{p(x-1)}$$

where A and P are constants. The graph of Iny against x is a straight line passing through the points $(2;\ 1.60)$ and $(5; 2.92)$, as shown in the diagram. Find the values of A and p correct to 2 significant figures. **[5]**

Suggested Solution

$y = Ae^{p(x-1)}$

$Iny = In\left[Ae^{p(x-1)}\right]$

$Iny = In(A) + In\left[e^{p(x-1)}\right]$

$Iny = InA + p(x-1)$

$Iny = InA + px - p$

$Iny = px + (InA - p)$

$\Rightarrow Gradient = p$ and $Vertical\ axis\ intercept = InA - p$

Now:

$$Gradient = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2.92 - 1.60}{5 - 2}$$

$$= \frac{1.32}{3}$$

$$= \frac{11}{25}$$

$$= 0.44$$

$\therefore p = 0.44$ (to 2 $s.f.$)

Finding the vertical intercept using $(5; 2.92)$ and $(0; c)$:

NB: You can also use the point $(2;\ 1.60)$

Now:

$$Gradient = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\Rightarrow \frac{c - 2.92}{0 - 5} = 0.44$$

$$\Rightarrow \frac{c - 2.92}{-5} = 0.44$$

$$\Rightarrow c - 2.92 = 0.44 \times -5$$

$$\Rightarrow c - 2.92 = -2.2$$

$$\Rightarrow c = -2.2 + 2.92$$

$$\therefore c = 0.72$$

Now:

$$InA - p = 0.72$$

$$InA = 0.72 + p$$

$$InA = 0.72 + 0.44$$

$InA = 1.16$

$\Rightarrow A = e^{1.16}$

$A = 3.189933276$

$\therefore A = 3.2$ (to 2 $s.f.$)

PAST EXAMINATION QUESTIONS

ZIMSEC NOVEMBER 1997 PAPER

The relation between the variable quantities x and y is

$$y = a + \frac{b}{x}$$

where a and b are constants. Four pairs of corresponding values for x and y, measured experimentally are shown in the following table

x	1.0	1.5	2.6	4.3
y	4.9	4.3	3.8	3.5

By plotting a graph of y against $\frac{1}{x}$, determine values for a and b. **[5]**

Answer: $a = 3.1$ and $b = 1.8$

ZIMSEC JUNE 2004 PAPER 1

The variables x and y are believed to satisfy a relationship of the form $y = ab^x$, where a and b are constants. The values obtained in an experiment are shown in the table below

x	1	2	3	4	5
y	14.1	15.8	17.8	19.9	22.4

By plotting a suitable graph, verify the relationship. Use your graph to calculate approximate values of a and b. **[7]**

Answer: $a = 13$ and $b = 1.1$

ZIMSEC NOVEMBER 2004 PAPER 1

The table below shows values of two quantities P and Q connected by the equation

$$P = a + be^{-Q}$$

Q	-2.485	-2.303	-1.792	-1.386	-0.693
P	0	1	3	4	5

By means of a suitable graph, find the values of a and b. **[5]**

Answer: $a = 6$ and $b = -\frac{1}{2}$

ZIMSEC JUNE 2005 PAPER 1

The variable p and q are known to be connected by the formula $pq^M = N$, where M and N are constants. In an experiment, the following values of p and q were obtained:

p	1.5	2	3	4	5
q	5.05	4.22	3.27	2.73	2.38

(i) Construct a table of values of lg$p$ and lg$q$, giving the values to 2 decimal places. **[2]**

(ii) Plot the values of lg$p$ against the values of lg$q$ using a scale of $2cm$ to represent 0.1 units on each axis. **[2]**

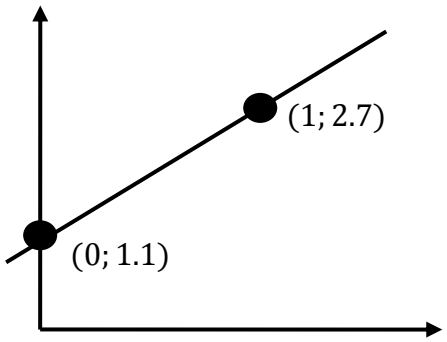
(iii)Hence

(a) show that the given values are in agreement with the formula $pq^M = N$, **[2]**

(b) determine the the values of M and N. **[4]**

ZIMSEC JUNE 2010 PAPER 1

A mathematician working with an exponential relation $y = ab^x$ reduced it to linear form and came out with the graph shown in the diagram below



(i) State the label on each of the axes **[2]**

(ii) Calculate the value of a and the value of b. **[3]**

Answer: Iny(y −axis); x(x −axis): $a = 3.004$ and $b = 4.953$

ZIMSEC JUNE 2013 PAPER 1

The variable quantities x and y are related by the equation $y = ab^x$ where a and b are constants. When the graph is plotted showing values of Iny on the vertical axis and values of x on the horizontal axis, the points lie on a straight line having gradient 0.8 and y intercept at 3.3.

Find the values of a and the b correct to one decimal place. **[4]**

Answer: $a = 27.1$ and $b = -2.2$

ZIMSEC NOVEMBER 2016 PAPER 1

The variable quantities x and y are related by the rule $y = \frac{P}{\sqrt{x}}$, where P is a constant. Five pairs of values x and y measured experimentally are shown in the table.

x	20	30	45	50	75
y	0.9	1.1	1.35	1.40	1.52

(i) Plot the graph of y against $\frac{1}{\sqrt{x}}$. **[3]**

(ii) Use the graph to find the value of P. **[2]**

Answer: $P = 7.2$

CAMBRIDGE QUESTIONS

CAMBRIDGE JUNE 2007 PAPER 2

The variables x and y satisfy the relation $3^y = 4^{x+2}$

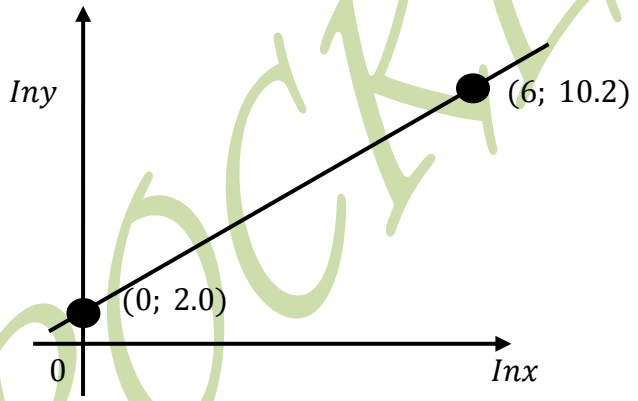
(i) By taking logarithms, show that the graph of y against x. Find the exact value of the gradient of this line. **[3]**

(ii) Calculate the x-coordinate of the point of intersection of this line with the line $y = 2x$, giving your answer correct to 2 decimal places. **[3]**

Answer: (i) $yIn3 = (x+2)In4 \; or \; ylog3 = (x+2)log4; Gradient = \frac{In4}{In3}$ or $\frac{log4}{log3}$

(ii) $x = 3.42$

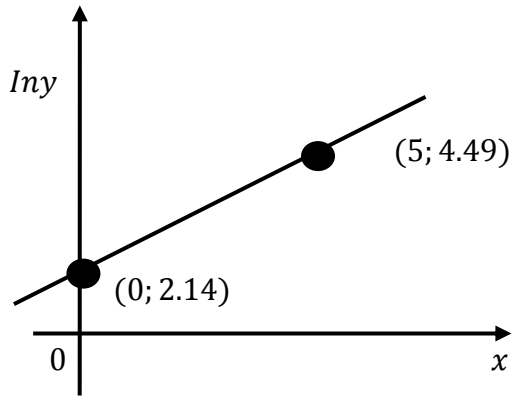
CAMBRIDGE JUNE 2011 PAPER 2



The variables x and y satisfy the equation $y = Kx^m$, where K and m are constants. The graph of Iny against Inx is a straight line passing through the points $(0;\ 2.0)$ and $(6;\ 10.2)$, as shown in the diagram. Find the values of K and m correct to 2 decimal places. **[5]**

Answer: $K = 7.39$ and $m = 1.37$

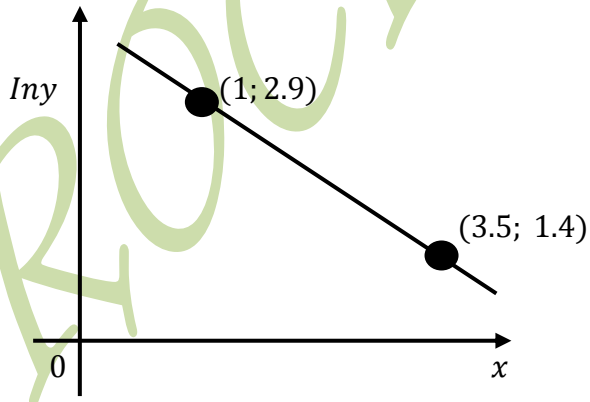
CAMBRIDGE JUNE 2012 PAPER 2



The variables x and y satisfy the equation $y = A(b^x)$, where A and b are constants. The graph of Iny against x is a straight line passing through the points $(0;\ 2.14)$ and $(5;\ 4.49)$ as shown in the diagram. Find the values of A and b correct to 1 decimal place. **[5]**

Answer: $A = 8.5$ and $b = 1.6$

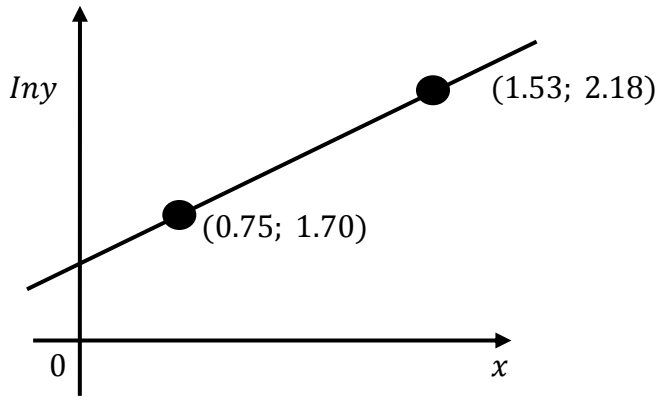
CAMBRIDGE NOVEMBER 2012 PAPER 2



The variables x and y satisfy the equation $y = A(b^{-x})$, where A axd b are constants. The graph of Iny against x is a straight line passing through the points $(1;\ 2.9)$ and $(3.5;\ 1.4)$, as shown in the diagram. Find the values of A and b correct to 2 decimal places. **[6]**

Answer: $A = 33.12$ and $b = 1.82$

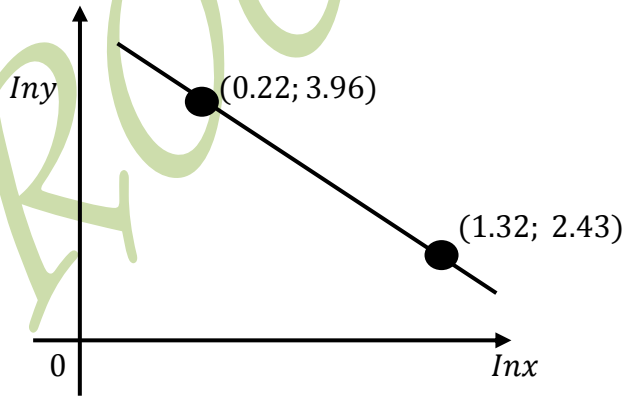
CAMBRIDGE NOVEMBER 2014 PAPER 2



The variables x and y satisfy the equation $y = A(b^x)$, where A and b are constants. The graph of Iny against x is a straight line passing through the points $(0.75;\ 1.70)$ and $(1.53;\ 2.18)$, as shown in the diagram. Find the values of A and b correct to 2 decimal places. **[5]**

Answer: $A = 3.45$ and $b = 1.85$

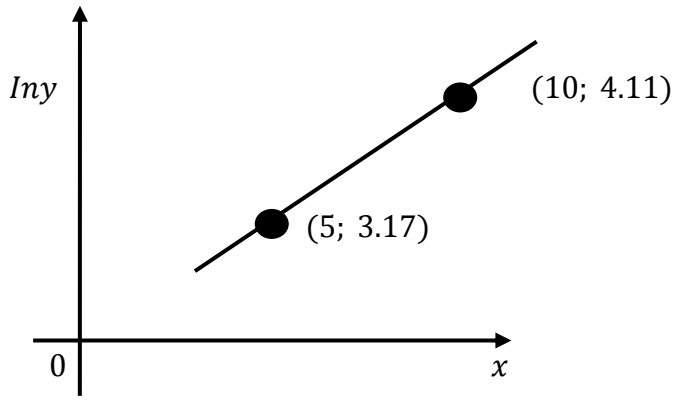
CAMBRIDGE NOVEMBER 2015 PAPER 2



The variables x and y satisfy the equation $y = Kx^m$, where k axd a are constants. The graph of Iny against Inx is a straight line passing through the points $(0.22;\ 3.96)$ and$(1.32;\ 2.43)$, as shown in the diagram. Find the values of K and m correct to 2 significant figures. **[5]**

Answer: $K = 71$ or 72 and $m = -1.4$

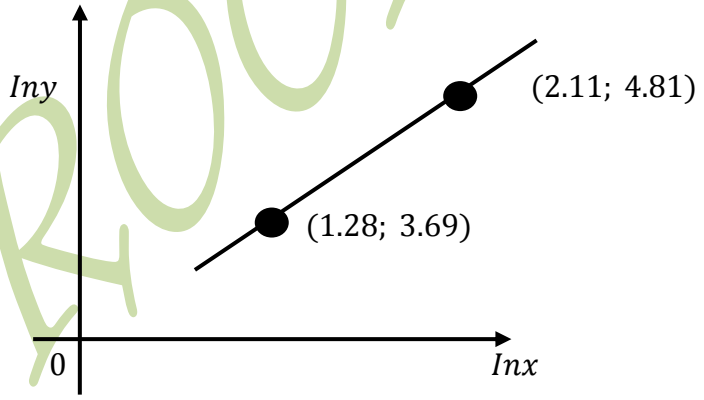
CAMBRIDGE NOVEMBER 2016 PAPER 21



The variables x and y satisfy the equation $y = Ae^{px}$, where A and p are constants. The graph of Iny against x is a straight line passing through the points $(5;\ 3.17)$ and$(10;\ 4.77)$, as shown in the diagram. Find the values of A and p correct to 2 decimal places. **[5]**

Answer: $A = 4.81$ and $p = 0.32$

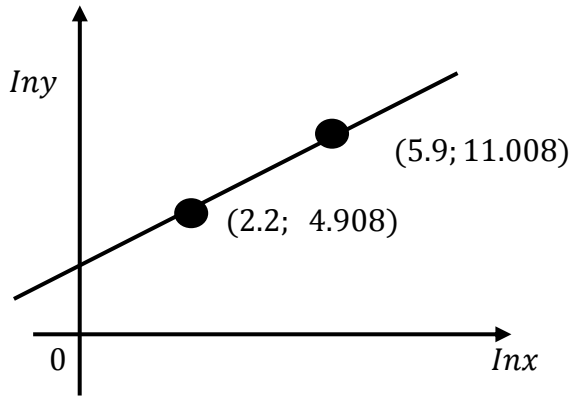
CAMBRIDGE NOVEMBER 2016 PAPER 23



The variables x and y satisfy the equation $y = Kx^p$, where K and p are constants. The graph of Iny against Inx is a straight line passing through the points $(1.28;\ 3.69)$ and$(2.11;\ 4.81)$, as shown in the diagram. Find the values of K and p correct to 2 decimal places. **[5]**

Answer: $K = 7.11$ or 7.12 and $p = 1.35$

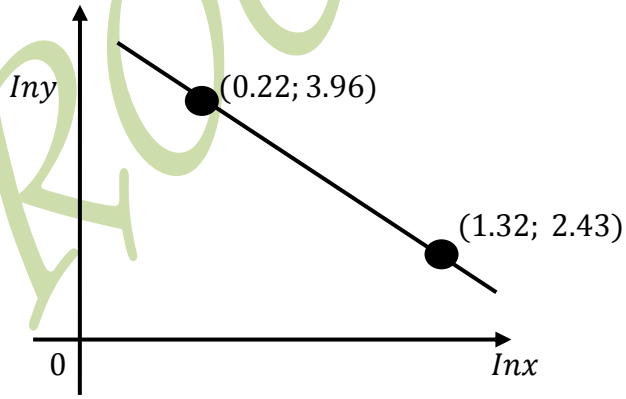
CAMBRIDGE JUNE 2018 PAPER 2



The variables x and y satisfy the equation $y = A \times B^{Inx}$, where A and B are constants. The graph of Iny against Inx is a straight line passing through the points $(2.2;\ 4.908)$ and $(5.9;\ 11.008)$ as shown in the diagram. Find the values of A and B correct to 2 significant figures. **[5]**

Answer: $A = 3.6$ and $B = 5.2$

CAMBRIDGE NOVEMBER 2019 PAPER 2



The variables x and y satisfy the equation $y = kx^a$, where k and a are constants. The graph of Iny against Inx is a straight line passing through the points $(0.22;\ 3.96)$ and$(1.32;\ 2.43)$, as shown in the diagram. Find the values of k and a correct to 3 significant figures. **[5]**

Answer: $a = -1.39$ and $k = 71.2$

ASANTE SANA

*******THERE IS A LIGHT AT THE END OF EVERY TUNNEL *******

CONSTRUCTIVE COMMENTS ON THE FORM OF THE PRESENTATION, INCLUDING ANY OMISSIONS OR ERRORS, ARE WELCOME.

ENJOY

Nyasha P. Tarakino (Trockers)

+263772978155/+263717267175

ntarakino@gmail.com