The Method of Differences

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30 JUNE 2020

SYLLABUS (6042) REQUIREMENTS

- use the method of differences to obtain the sum of finite series

The Method of Differences

- When working with sequences and series, sometimes partial fractions are needed to solve the problem.
- The first step is to recognize what types of sequence or series problems require the use of partial fractions.
- Solving the partial fraction will help set up a new series that enables to solve using limits.
- The first two examples require the use of partial fractions.

Solved Problems

Question 1

Find

$$\sum_{r=1}^{n} \frac{1}{(r+2)(r+3)}$$

Suggested Solution

Step 1

Express $\frac{1}{(r+2)(r+3)}$ into Partial Fractions.

Let $\frac{1}{(r+2)(r+3)} \equiv \frac{A}{(r+2)} + \frac{B}{(r+3)}$

$\Rightarrow A(r+3) + B(r+2) = 1$

When $r = -3$; $-B = 1 \Rightarrow B = -1$.

When $r = -2$; $A = 1$

NB: Sum of constants should give zero i.e. $A + B = 0$

$$\therefore \frac{1}{(r+2)(r+3)} \equiv \frac{1}{(r+2)} - \frac{1}{(r+3)}$$

Step 2

- Write out the first few terms of the series and notice what is happening with the values.
- The second value is always cancelling with the exception of the last term, along with the first value after the initial term.
- This is a Telescoping Series.

$$r = 1 \quad r = 2 \quad r = 3 \quad r = 4 \quad \ldots \quad r = n$$

$$\therefore \sum_{r=1}^{n} \frac{1}{(r+2)(r+3)} \equiv \sum_{r=1}^{n} \frac{1}{(r+2)} - \frac{1}{(r+3)}$$

Now

$$\sum_{r=1}^{n} \frac{1}{(r+2)} - \frac{1}{(r+3)} =$$

$$\left[\frac{1}{3} - \cancel{\frac{1}{4}}\right] + \qquad (\text{for } r = 1)$$

$$\left[\cancel{\frac{1}{4}} - \frac{1}{5}\right] + \qquad (\text{for } r = 2)$$

$$\left[\frac{1}{5} - \frac{1}{6}\right] + \qquad (\text{for } r = 3)$$

$$\ldots +$$

$$\left[\frac{1}{n+1} - \cancel{\frac{1}{n+2}}\right] + \qquad (\text{for } r = n - 1)$$

$$\left[\cancel{\frac{1}{n+2}} - \frac{1}{n+3}\right] \qquad (\text{for } r = n)$$

- After performing relevant cancelling (See the illustration on the 1st two and the last two lines) we should obtain

$$\therefore \sum_{r=1}^{n} \frac{1}{(r+2)} - \frac{1}{(r+3)} \equiv \frac{1}{3} - \frac{1}{n+3}.$$

Question 2

Find if the series converges or diverges. If convergent then find its sum.

$$\sum_{r=1}^{\infty} \frac{1}{(r+2)(r+3)}$$

Suggested Solution

$$\sum_{r=1}^{\infty} \frac{1}{(r+2)} - \frac{1}{(r+3)} \equiv \frac{1}{3} - \frac{1}{n+3}$$

$$= Lim_{n\to\infty} \left[\frac{1}{3} - \frac{1}{n+3}\right] = \frac{1}{3} - \frac{1}{\infty} = \frac{1}{3}.$$

- Since it is possible to solve for the limit, it is known that the series converges, and the sum of the series is $\frac{1}{3}$.

$$\therefore \sum_{r=1}^{\infty} \frac{1}{(r+2)} - \frac{1}{(r+3)} \equiv \frac{1}{3}$$

Question 3

ZIMSEC November 2018 Paper 2

a) Find

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)}$$

[6]

b) Hence, or otherwise deduce the value of

$$\sum_{r=1}^{\infty} \frac{2}{r(r+1)(r+2)}$$

[2]

Suggested Solution

a)

Step 1

Express $\frac{2}{r(r+1)(r+2)}$ into Partial Fractions.

Let $\frac{2}{r(r+1)(r+2)} \equiv \frac{A}{r} + \frac{B}{(r+1)} + \frac{C}{(r+2)}$

$\Rightarrow A(r+1)(r+2) + B(r)(r+2) + C(r)(r+1) = 2$

When $r = 0$; $2A = 2 \Rightarrow A = 1$

When $r = -1$; $-B = 2 \Rightarrow B = -2$

When $r = -2$; $2C = 2 \Rightarrow C = 1$

NB: Sum of constants should give zero i.e. $A + B + C = 0$

$\therefore \frac{2}{r(r+1)(r+2)} \equiv \frac{1}{r} - \frac{2}{(r+1)} + \frac{1}{(r+2)}$

Step 2

- Express terms as differences using Step 1 and we should do this for at least first 4 terms and at least last 3 terms so that we can show relevant cancelling clearly.

$i.e.$ for $r=1 \quad r=2 \quad r=3 \quad r=4 \quad r=5 \quad \dots \; r=n-2 \quad r=n-1 \quad r=n$

$$\therefore \sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} \equiv \sum_{r=1}^{n} \frac{1}{r} - \frac{2}{(r+1)} + \frac{1}{(r+2)}$$

Now

$$\sum_{r=1}^{n} \frac{1}{r} - \frac{2}{(r+1)} + \frac{1}{(r+2)} =$$

$$\left[\frac{1}{1} - \frac{2}{2} + \frac{1}{3}\right] + \quad (\text{for } r=1)$$

$$\left[\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right] + \quad (\text{for } r=2)$$

$$\left[\frac{1}{3} - \frac{2}{4} + \frac{1}{5}\right] + \quad (\text{for } r=3)$$

$$\left[\frac{1}{4} - \frac{2}{5} + \frac{1}{6}\right] + \quad (\text{for } r=4)$$

$$\left[\frac{1}{5} - \frac{2}{6} + \frac{1}{7}\right] + \quad (\text{for } r=5)$$

$$\dots +$$

$$\left[\frac{1}{n-2} - \frac{2}{n-1} + \frac{1}{n}\right] + \quad (\text{for } r=n-2)$$

$$\left[\frac{1}{n-1} - \frac{2}{n} + \frac{1}{n+1}\right] + \quad (\text{for } r=n-1)$$

$$\left[\frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2}\right] \quad (\text{for } r=n)$$

- After performing relevant cancelling (See the illustration on the 1st three and the last three lines and also observe the matching colours) we should obtain

$$\sum_{r=1}^{n} \frac{1}{r} - \frac{2}{(r+1)} + \frac{1}{(r+2)} = \frac{1}{2} + \frac{1}{n+1} - \frac{2}{n+1} + \frac{1}{n+2}$$

$$= \frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$$

$$\therefore \sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} = \frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2} \text{ or } \frac{1}{2} - \frac{1}{(n+1)(n+2)}$$

b) $$\sum_{r=1}^{\infty} \frac{2}{r(r+1)(r+2)} \equiv \frac{1}{2} - \frac{1}{(n+1)(n+2)}$$

$$\Rightarrow Lim_{n\to\infty} \left[\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right] = \frac{1}{2} - \frac{1}{\infty} = \frac{1}{2}.$$

NB: $\frac{1}{\infty} = 0$

$\therefore$ The value of $\sum_{r=1}^{\infty} \frac{2}{r(r+1)(r+2)} = \frac{1}{2}$

Question 4

Show that

$$\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)} \equiv \frac{2}{r(r+1)(r+2)}$$

Hence, or otherwise find a simplified expression for

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)}$$

NOTES

- There is no need for partial fractions as we must show the difference between the two fractions is the SINGLE fraction given.
- Partial fractions would be needed if working the other way round.
- Show that the single fraction can be written as the difference between the two fractions

Suggested Solution

a)

$$\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)} = \frac{1(r+2) - 1(r)}{r(r+1)(r+2)}$$

$$= \frac{r+2-r}{r(r+1)(r+2)}$$

$$= \frac{2}{r(r+1)(r+2)} \quad (as\ required)$$

b)

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} \equiv \sum_{r=1}^{n} \frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}$$

Notes

- Write out the first few terms of the series and notice what is happening with the values.
- The second value is always cancelling with the exception of the last term, along with the first value after the initial term.
- This is a Telescoping Series.

$$r = 1 \quad r = 2 \quad r = 3 \quad r = 4 \quad \dots \quad r = n$$

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)} =$$

$$\left[\frac{1}{(1)(2)} - \frac{1}{(2)(3)}\right] + \qquad (\text{for } r = 1)$$

$$\left[\frac{1}{(2)(3)} - \frac{1}{(3)(4)}\right] + \qquad (\text{for } r = 2)$$

$$\left[\frac{1}{(3)(4)} - \frac{1}{(4)(5)}\right] + \qquad (\text{for } r = 3)$$

$$\ldots +$$

$$+\left[\frac{1}{(n-1)(n)} - \frac{1}{(n)(n+1)}\right] \qquad (\text{for } r = n-1)$$

$$+\left[\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)}\right] \qquad (\text{for } r = n)$$

- After performing relevant cancelling (See the illustration on the 1st two and the last two lines and also observe the matching colours) we should obtain

Now

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)}$$

Question 5

Cambridge June 2006 Further Mathematics Paper 1

Given that for all the real values of r

$$(2r+1)^3 - (2r-1)^3 = Ar^2 + B$$

where A and B are constants.

a) Find the value of A and the value of B. **[2]**

b) Hence or otherwise prove that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1) \qquad [\mathbf{5}]$$

c) Calculate

$$\sum_{r=1}^{40} (3r-1)^2 \qquad [\mathbf{2}]$$

Suggested Solution

a)

$$(2r+1)^3 = (2r)^3 + (3)(2r)^2(1) + \frac{(3)(2)(2r)(1)^2}{2!} + \frac{(3)(2)(1)(2r)^0(1)^3}{3!}$$

$$= 8r^3 + 12r^2 + 6r + 1$$

$$(2r-1)^3 = (2r)^3 + (3)(2r)^2(-1) + \frac{(3)(2)(2r)(-1)^2}{2!} + \frac{(3)(2)(1)(2r)^0(-1)^3}{3!}$$

$$= 8r^3 - 12r^2 + 6r - 1$$

$$(2r+1)^3 - (2r-1)^3 = (8r^3 + 12r^2 + 6r + 1) - (8r^3 - 12r^2 + 6r - 1)$$

$$= 24r^2 + 2$$

$\therefore A = 24$ and $B = 2$

b) $(2r+1)^3 - (2r-1)^3 = 24r^2 + 2$

$$\Rightarrow r^2 = \frac{(2r+1)^3 - (2r-1)^3 - 2}{24}$$

Now:

$$\sum_{r=1}^{n} r^2 = \sum_{r=1}^{n} \frac{(2r+1)^3 - (2r-1)^3 - 2}{24} =$$

$$\left[\frac{(3)^3 - (1)^3 - 2}{24}\right] + \qquad (\text{for } r = 1)$$

$$\left[\frac{(5)^3 - (3)^3 - 2}{24}\right] + \qquad (\text{for } r = 2)$$

$$\left[\frac{(7)^3 - (5)^3 - 2}{24}\right] + \qquad (\text{for } r = 3)$$

$\dots +$

$$+\left[\frac{(2n-1)^3 - (2n-3)^3 - 2}{24}\right] \qquad (\text{for } r = n-1)$$

$$+\left[\frac{(2n+1)^3 - (2n-1)^3 - 2}{24}\right] \qquad (\text{for } r = n)$$

- After performing relevant cancelling (Observe the matching colours) we should obtain

$$\sum_{r=1}^{n} \frac{(2r+1)^3 - (2r-1)^3 - 2}{24} = \frac{-(1)^3 + (2n+1)^3 - 2n}{24}$$

$$= \frac{8n^3 + 12n^2 + 6n - 2n}{24}$$

$$= \frac{8n^3 + 12n^2 + 4n}{24}$$

$$= \frac{4n(2n^2 + 3n + 1)}{24}$$

$$= \frac{4n(2n^2 + 2n + n + 1)}{24}$$

$$= \frac{4n[2n(n + 1) + 1(n + 1)]}{24}$$

$$= \frac{4n(2n + 1)(n + 1)}{24}$$

$$= \frac{1}{6}n(n + 1)(2n + 1) \ \ (as\ required)$$

c) $\sum_{r=1}^{40}(3r - 1)^2 = \sum_{r=1}^{40}(9r^2 - 6r + 1)$

$$= 9\sum_{r=1}^{40}r^2 - 6\sum_{r=1}^{40}r + \sum_{r=1}^{40}1$$

$$= 9\left[\frac{1}{6}40(40 + 1)(2 \times 40 + 1)\ \right] - 6\left[\frac{1}{2}40(40 + 1)\ \right] + (40)(1)$$

$$= 9\left[\frac{132\,840}{6}\ \right] - 6\left[\frac{1\,640}{2}\right] + 40$$

$$= 199\,260 - 4\,920 + 40$$

$$= 194\,380$$

NB:

1) $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n + 1)(2n + 1)$

2) $\sum_{r=1}^{n} r = \frac{1}{2}n(n + 1)$

3) $\sum_{r=1}^{n} a = na$

Question 6

Cambridge Further Mathematics Paper 1

Find

$$\sum_{r=1}^{n} \frac{r+4}{r(r+1)(r+2)}$$

Suggested Solution

Let $\frac{r+4}{r(r+1)(r+2)} \equiv \frac{A}{r} + \frac{B}{r+1} + \frac{C}{r+2}$

$$\frac{r+4}{r(r+1)(r+2)} \equiv \frac{A(r+1)(r+2) + Br(r+2) + Cr(r+1)}{r(r+1)(r+2)}$$

$$r + 4 \Rightarrow A(r+1)(r+2) + Br(r+2) + Cr(r+1)$$

When $r = 0$: $4 = 2A$ $\therefore A = 2$

When $r = -1$: $3 = -B$ $\therefore B = -3$

When $r = -2$: $2 = 2C$ $\therefore C = 1$

$$\therefore \frac{r+4}{r(r+1)(r+2)} \equiv \frac{2}{r} - \frac{3}{r+1} + \frac{1}{r+2}$$

Now

$$\sum_{r=1}^{n} \frac{r+4}{r(r+1)(r+2)} \equiv \sum_{r=1}^{n} \frac{2}{r} - \frac{3}{r+1} + \frac{1}{r+2}$$

When $r = 1$: $\left(\frac{2}{1} - \frac{3}{2} + \frac{1}{3}\right) +$

$r = 2$: $\left(\frac{2}{2} - \frac{3}{3} + \frac{1}{4}\right) +$

$r = 3$: $\left(\frac{2}{3} - \frac{3}{4} + \frac{1}{5}\right) +$

$r = 4$: $\left(\frac{2}{4} - \frac{3}{5} + \frac{1}{6}\right) +$

$r = 5$: $\left(\frac{2}{5} - \frac{3}{6} + \frac{1}{7}\right) +$

.
.
.

$r = n-1$: $\left(\frac{2}{n-1} - \frac{3}{n} + \frac{1}{n+1}\right) +$

$r = n-2$: $\left(\frac{2}{n-2} - \frac{3}{n-1} + \frac{1}{n}\right) +$

$r = n-1$: $\left(\frac{2}{n-1} - \frac{3}{n} + \frac{1}{n+1}\right) +$

$r = n$: $\left(\frac{2}{n} - \frac{3}{n+1} + \frac{1}{n+2}\right)$

Now:

$$\sum_{r=1}^{n} \frac{2}{r} - \frac{3}{r+1} + \frac{1}{r+2} = \frac{2}{1} - \frac{3}{2} + \frac{2}{2} + \frac{1}{n+1} - \frac{3}{n+1} + \frac{1}{n+2}$$

$$= \frac{3}{2} - \frac{2}{n+1} + \frac{1}{n+2}$$

Hence deduce the value of

$$\sum_{r=1}^{\infty} \frac{r+4}{r(r+1)(r+2)}$$ [2]

<u>Suggested Solution</u>

$$\sum_{r=1}^{\infty} \frac{r+4}{r(r+1)(r+2)} = \lim_{n\to\infty}\left(\frac{3}{2} - \frac{2}{n+1} + \frac{1}{n+2}\right)$$

$$=\frac{3}{2}-\frac{2}{\infty}+\frac{1}{\infty}$$

$$=\frac{3}{2}-0+0$$

$$=\frac{3}{2}$$

Question 7

Cambridge Further Mathematics Paper 1

Simplify $\frac{1}{r!}-\frac{1}{(r+1)!}$.

Suggested Solution

$$\frac{1}{r!}-\frac{1}{(r+1)!}\equiv\frac{(r+1)!-r!}{r!\,(r+1)!}$$

$$\equiv\frac{[(r+1)\times r\times r-1\times\ldots\times 1]-[r\times r-1\times\ldots\times 1]}{r!\,(r+1)!}$$

$$\equiv\frac{[r\times r-1\times\ldots\times 1]\{(r+1)-(1)\}}{r!\,(r+1)!}$$

$$\equiv\frac{r!\,\{r\}}{r!\,(r+1)!}$$

$$\equiv\frac{r}{(r+1)!}$$

Hence find

$$\sum_{r=1}^{n}\frac{r}{(r+1)!}$$

Suggested Solution

$$\sum_{r=1}^{n}\frac{r}{(r+1)!}\equiv\sum_{r=1}^{n}\frac{1}{r!}-\frac{1}{(r+1)!}$$

When: $r = 1$: $\left(\frac{1}{1!} - \cancel{\frac{1}{2!}}\right) +$

$r = 2$: $\left(\cancel{\frac{1}{2!}} - \cancel{\frac{1}{3!}}\right) +$

$r = 3$: $\left(\cancel{\frac{1}{3!}} - \cancel{\frac{1}{4!}}\right) +$

.
.
.

$r = n - 1$: $\left(\cancel{\frac{1}{n-1!}} - \cancel{\frac{1}{n!}}\right) +$

$r = n$: $\left(\cancel{\frac{1}{n!}} - \frac{1}{(n+1)!}\right)$

Now:

$$\sum_{r=1}^{n} \frac{r}{(r+1)!} \equiv 1 - \frac{1}{(n+1)!}$$

Question 8

Cambridge Further Mathematics Paper 1

Given that $\frac{1}{r(r+1)(r+2)} = \frac{1}{2r(r+1)} - \frac{1}{2(r+1)(r+2)}$ find

$$\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}$$

Suggested Solution

$$\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} \equiv \sum_{r=1}^{n} \frac{1}{2r(r+1)} - \frac{1}{2(r+1)(r+2)}$$

When $r = 1$: $\left(\frac{1}{2(1)(2)} - \frac{1}{2(2)(3)}\right) +$

$r = 2$: $\left(\frac{1}{2(2)(3)} - \frac{1}{2(3)(4)}\right) +$

$r = 3$: $\left(\frac{1}{2(3)(4)} - \frac{1}{2(4)(5)}\right) +$

.
.
.

$r = n - 1$: $\left(\frac{1}{2(n-1)(n)} - \frac{1}{2(n)(n+1)}\right) +$

$r = n$: $\left(\frac{1}{2(n)(n+1)} - \frac{1}{2(n+1)(n+2)}\right)$

$$\therefore \sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$$

Hence deduce the value of

$$\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)}$$

Answer

$$\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)} = \lim_{n\to\infty} \left(\frac{1}{4} - \frac{1}{2(n+1)(n+2)}\right)$$

$$= \frac{1}{4} - \frac{1}{\infty}$$

$$= \frac{1}{4} - 0$$

$$= \frac{1}{4}$$

Question 10

Cape Pure Mathematics Unit 2

a) Show that

$$1 \equiv \frac{1}{2}[r(r+1) - r(r-1)]$$

b) Hence show using the method of differences that

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

c) Evaluate

$$\sum_{r=10}^{30} 4r$$

Suggested Solution

a) $1 \equiv \frac{1}{2}[r(r+1) - r(r-1)]$

LHS:

$$\frac{1}{2}[r(r+1) - r(r-1)] = \frac{1}{2}(r^2 + r - r^2 + r)$$

$$= \frac{1}{2}(2r)$$

$$= r \text{ (As required)}$$

b)

$$\sum_{r=1}^{n} r = \sum_{r=1}^{n} \frac{1}{2}[r(r+1) - r(r-1)] =$$

$\frac{1}{2}[1(2) - 1(0)] +$ (for $r = 1$)

$\frac{1}{2}[2(3) - 2(1)] +$ (for $r = 2$)

$\frac{1}{2}[3(4) - 3(2)] +$ (for $r = 3$)

$\ldots +$

$$+\frac{1}{2}[(n-1)(n)-(n-1)(n-2)] \qquad (\text{for } r=n-1)$$

$$+\frac{1}{2}[n(n+1)-n(n-1)] \qquad (\text{for } r=n)$$

- After performing relevant cancelling (Observe the matching colours) we should obtain

$$\sum_{r=1}^{n} r=\frac{1}{2}n(n+1)$$

c) $$\sum_{r=10}^{30} 4r = 4\sum_{r=1}^{30} r-4\sum_{r=1}^{9} r$$

$$=4\left[\frac{1}{2}30(30+1)\ \right]-4\left[\frac{1}{2}9(9+1)\ \right]$$

$$=4[15(31)\ ]-4[9(5)\ ]$$

$$=1\,860-180$$

$$=1\,680$$

PRACTICE QUESTIONS

QUESTION 1

a) Show that $\frac{1}{r}-\frac{3}{r+1}+\frac{2}{r+2}\equiv\frac{2-r}{r(r+1)(r+2)}$

b) Hence show that

$$\sum_{r=1}^{n}\frac{2-r}{r(r+1)(r+2)}=\frac{n}{(n+1)(n+2)}$$

c) Find the value of

$$\sum_{r=2}^{\infty}\frac{2-r}{r(r+1)(r+2)}$$

QUESTION 2

a) Show that $\frac{1}{3r-1}-\frac{1}{3r+2}\equiv\frac{3}{(3r-1)(3r+2)}$

b) Hence show that

$$\sum_{r=1}^{2n}\frac{1}{(3r-1)(3r+2)}=\frac{n}{2(3n+1)}$$

QUESTION 3

a) Show that $\frac{1}{r}-\frac{1}{r+2}\equiv\frac{2}{r(r+2)}$

b) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n}\frac{2}{r(r+2)}$$

c) Given that

$$\sum_{r=N+1}^{\infty}\frac{2}{r(r+2)}=\frac{11}{30},\text{ find the vaue of } N.$$

QUESTION 4

a) Show that $\frac{1}{r-1}-\frac{1}{r+1}\equiv\frac{2}{r^2-1}$

b) Hence find an expression, in terms of n, for

$$\sum_{r=2}^{n}\frac{2}{r^2-1}$$

c) Find the value of

$$\sum_{r=1000}^{\infty}\frac{2}{r^2-1}$$

QUESTION 5

a) Show that $\frac{1}{r^2}-\frac{1}{(r+1)^2}\equiv\frac{2r+1}{r^2(r+1)^2}$

b) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n}\frac{2r+1}{r^2(r+1)^2}$$

c) Find the value of

$$\sum_{r=2}^{\infty}\frac{2r+1}{r^2(r+1)^2}$$

QUESTION 6

a) Show that $\frac{1}{2r-3}-\frac{1}{2r+1}\equiv\frac{4}{4r^2-4r-3}$

b) Hence find an expression, in terms of n, for

$$\sum_{r=2}^{n}\frac{4}{4r^2-4r-3}$$

c) Show that

$$\sum_{r=2}^{\infty} \frac{4}{4r^2 - 4r - 3} = \frac{4}{3}$$

QUESTION 7

a) Show that $\frac{2}{r} - \frac{1}{r+1} - \frac{1}{r+2} \equiv \frac{3r+4}{r(r+1)(r+2)}$

b) Hence:

(i) Find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{3r+4}{r(r+1)(r+2)}$$

(ii) Write down the value of

$$\sum_{r=1}^{\infty} \frac{3r+4}{r(r+1)(r+2)}$$

c) Given that

$$\sum_{r=N+1}^{\infty} \frac{3r+4}{r(r+1)(r+2)} = \frac{7}{10}, \text{ find the vaue of } N.$$

QUESTION 8

a) Show that $\frac{1}{r!} - \frac{1}{(r+1)!} \equiv \frac{r}{(r+1)!}$

b) Hence find an expression, in terms of n, for

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{r}{(r+1)!}$$

QUESTION 9

ZIMSEC November 2019 Paper 2

a) Express $\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}$ in the form $\frac{A}{r(r+1)(r+2)}$ **[2]**

b) Hence show that

$$\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)} \qquad [5]$$

c) Deduce the value of

$$\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)}. \qquad [2]$$

QUESTION 10

ZIMSEC November 2019 Additional Mathematics Paper 1

a) Show that

$$\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{3}{4} - \frac{2n+3}{2(n+1)(n+2)} \qquad [7]$$

b) Hence, or otherwise, find

$$\sum_{r=1}^{\infty} \frac{1}{r(r+2)}. \qquad [1]$$

QUESTION 11

Cambridge January 2003 Further Mathematics Paper 4

a) Express $\frac{2}{(r+1)(r+3)}$ in partial fractions. **[2]**

b) Hence prove that

$$\sum_{r=1}^{n} \frac{2}{(r+1)(r+3)} = \frac{n(5n+13)}{6(n+2)(n+3)} \qquad [5]$$

QUESTION 12

Cambridge June 2019 Further Mathematics Paper 3

a) Use the method of differences to show that

$$\sum_{r=1}^{N} \frac{1}{(3r+1)(3r-2)} = \frac{1}{3} - \frac{1}{3(3N+1)}$$ **[4]**

b) Find the limit, as $N \to \infty$, of

$$\sum_{r=N+1}^{N^2} \frac{N}{(3r+1)(3r-2)}.$$ **[4]**

QUESTION 13

a) Show that $\frac{1}{r} - \frac{1}{r+1} \equiv \frac{1}{r(r+1)}$

b) Hence show that

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = 1 - \frac{1}{n+1}$$

c) Deduce that

$$\sum_{r=1}^{\infty} \frac{1}{r(r+1)} = 1$$

QUESTION 14

Given that $f(r) = \frac{1}{r(r+1)}$

a) Show that $f(r) - f(r+1) = \frac{a}{r(r+1)(r+2)}$, stating the value of a

b) Hence show that

$$\sum_{r=1}^{2n} \frac{1}{r(r+1)(r+2)} = \frac{n(2n+3)}{4(n+1)(2n+1)}$$

c) Deduce that

$$\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)}$$

ASANTE SANA

*******THERE IS A LIGHT AT THE END OF EVERY TUNNEL *******

CONSTRUCTIVE COMMENTS ON THE FORM OF THE PRESENTATION, INCLUDING ANY OMISSIONS OR ERRORS, ARE WELCOME.

ENJOY

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