Partial Fractions

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SYLLABUS (6042) REQUIREMENTS

- Expressing rational polynomials in partial fractions (including improper fractions)

Rational Fractions

Definition

It is a fraction of which both numerator and denominator are rational numbers or are polynomials

Types of Rational fractions

Proper Fraction

- A fraction where the numerator (top number) is less than the denominator (the bottom number
- The degree of the polynomial, say n , is the highest power occurring in the polynomial e.g. $x^2 - 3x + 1$ the degree is 2;
  $x^4 - 3x^3 + 1$ the degree is 4 and
  $2x - 8$ the degree is 1.
- A rational fraction is proper when the numerator has a degree which is at least one less than that of the denominator e.g.
  $\frac{2}{x+1}$; $\frac{8}{x^2-2x+1}$; $\frac{11}{2x^3+5x-2}$; $\frac{2x-1}{x^2-2x+1}$; $\frac{x^2+6x}{x^3+2x^2-x-3}$ etc

Improper Fraction

- A fraction where the numerator (top number) is greater than the denominator (the bottom number.
- A rational fraction is proper when the numerator has a degree which is greater than that of the denominator e.g.
  $\frac{2x-1}{5x+1}$; $\frac{x^2-2x+1}{3x^2+8x+11}$; $\frac{x^4+x-2}{x^2-2x+1}$; $\frac{2x^3+5x-2}{x^3+2x^2-x-3}$ etc

Changing improper fractions to proper fractions

Two methods can be employed to make fractions proper ones i.e. (i) Division algorithm

(ii) Algebraic Manipulation

Example

Make the expression $\frac{x+3}{x+5}$ a proper algebraic expression

Suggested Solution

Method 1: Algebraic Manipulation Algorithm

$$\frac{x+3}{x+5} = \frac{x+5-2}{x+5}$$

$$= \frac{x+5}{x+5} - \frac{2}{x+5}$$

$$= 1 - \frac{2}{x+5}$$

Method 2: Division Algorithm

$$\begin{array}{r|l} & \quad 1 \\ \hline x+5 & \quad x+3 \\ & - \; x+5 \\ \hline & \quad\; -2 \end{array}$$

$$\therefore \frac{x+3}{x+5} = 1 - \frac{2}{x+5}$$

Partial Fractions

Definition

- A technique for rewriting a rational function (i.e. a polynomial divided by another polynomial) as a sum of simpler rational functions.
- To express a single rational fraction into the sum of two or more single rational fractions is called Partial fraction resolution.

Types of Partial Fractions

There are basically 3 types of partial fractions namely: Linear, Repeated linear and Quadratic factors

Type	Denominator Containing	Expression	Form of Partial Fraction
1	Linear Factors	$\frac{f(x)}{(x+a)(x-b)\ldots(x+z)}$	$\frac{A}{(x+a)}+\frac{B}{(x-b)}+\cdots+\frac{Z}{(x+z)}$
2	Repeated Linear Factors	$\frac{f(x)}{(x+a)^n}$	$\frac{A}{(x+a)}+\frac{B}{(x+a)^2}+\cdots+\frac{C}{(x+a)^n}$
3	Quadratic Factors	$\frac{f(x)}{(ax^2+bx+c)(x+d)}$	$\frac{Ax+B}{(ax^2+bx+c)}+\frac{C}{(x+d)}$

Applications of Partial Fractions

Partial fractions are also used to solve problems involving: The Method of differences
Integration
Binomial expansion etc

Steps involved in resolving partial fractions

1. Make sure that the degree of the numerator is less than the degree of the denominator.
2. Factor the denominator as fully as possible.
3. Write the original rational function as a sum of fractions. Each of the terms in the sum gets one of the original factors as its denominator. The numerator of each new fraction is an arbitrary polynomial of degree one less than the degree of the denominator.
4. Use algebra to solve for the coefficients in the numerators of the new fractions.

What to do if the degree of the numerator is too large

Before we apply partial fractions, we have to apply the polynomial equivalent of long division, which works very much like long division for numbers.

Example of improper fractions

Illustration 1

Algebraic manipulation algorithm

$$\frac{x^2-2x}{x^2+8x+15}$$

$$\frac{x^2-2x}{x^2+8x+15} \equiv \frac{x^2+8x-10x+15-15}{x^2+8x+15}$$

$$\equiv \frac{x^2+8x+15\ -10x-15}{x^2+8x+15}$$

$$\equiv \frac{x^2+8x+15}{x^2+8x+15} - \frac{(10x+15)}{x^2+8x+15}$$

$$\equiv 1 - \frac{(10x+15)}{x^2+8x+15} \ \{\text{It is now proper}\}$$

NB: Algebraic manipulation is best when both the numerator and the denominator are of the same degree.

Illustration 2

Division algorithm - Polynomial long division

$$\frac{3x^3 - 2x^2 - 16x + 20}{x^2 - 4}$$

Divisor

Quotient

$$\begin{array}{r|l}
 & \quad 3x - 2 \\ \hline
x^2 - 4 & 3x^3 - 2x^2 - 16x + 20 \\
- & 3x^3 \qquad\quad - 12x \\ \hline
 & \qquad -2x^2 - 4x + 20 \\
- & \qquad -2x^2 \qquad\quad + 8 \\ \hline
 & \qquad\qquad\quad -4x + 12
\end{array}$$

Remainder

$$\therefore \frac{3x^3 - 2x^2 - 16x + 20}{x^2 - 4} \equiv 3x - 2 + \frac{(12 - 4x)}{x^2 - 4} \quad \{\text{It is now proper}\}$$

A. Linear Factors

- The denominators contain linear factors
- These are of the form

$$\frac{f(x)}{(x+a)(x-b)\dots(x+z)}$$

- This type of partial fractions can be resolved as

$$\frac{A}{(x+a)}+\frac{B}{(x-b)}+\cdots+\frac{Z}{(x+z)}$$

Worked Problems

Example 1

Express $\frac{3}{(x+2)(x-3)}$ in partial fractions

Suggested Solution

$$\frac{3}{(x+2)(x-3)}$$

Let $\frac{3}{(x+2)(x-3)} \equiv \frac{A}{x+2}+\frac{B}{x-3}$

$$\equiv \frac{A(x-3)+B(x+2)}{(x+2)(x-3)}$$

$\Rightarrow A(x-3)+B(x+2)=3$

When $x=3$: $5B=3$

$$\therefore B=\frac{3}{5}$$

When $x=-2$: $-5A=3$

$$\therefore A=-\frac{3}{5}$$

Now

$$\frac{3}{(x+2)(x-3)} \equiv \frac{3}{5(x-3)}-\frac{3}{5(x+2)}$$

Example 2

Convert $\frac{2x-3}{(x+2)(x-3)(2x+1)}$ into the sum of three partial fractions

Suggested Solution

$$\frac{2x-3}{(x+2)(x-3)(2x+1)}$$

Let $\frac{2x-3}{(x+2)(x-3)(2x+1)} \equiv \frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{2x+1}$

$$\equiv \frac{A(x-3)(2x+1) + B(x+2)(2x+1) + C(x+2)(x-3)}{(x+2)(x-3)(2x+1)}$$

$$\Rightarrow A(x-3)(2x+1) + B(x+2)(2x+1) + C(x+2)(x-3) = 2x-3$$

When $x = 3$: $(5)(7)B = 3$

$$\therefore B = \frac{3}{35}$$

When $x = -2$: $(-5)(-3)A = 3$

$$\therefore A = \frac{1}{5}$$

When $x = -\frac{1}{2}$: $\left(\frac{3}{2}\right)\left(-\frac{7}{2}\right)C = 3$

$$\therefore C = -\frac{4}{7}$$

Now

$$\frac{2x-3}{(x+2)(x-3)(2x+1)} \equiv \frac{1}{5(x+2)} + \frac{3}{35(x-3)} - \frac{4}{7(2x+1)}$$

Example 3

Resolve $\frac{3x^3-2x^2-16x+20}{x^2-4}$ into partial fractions

Suggested Solution

$$\frac{3x^3 - 2x^2 - 16x + 20}{x^2 - 4} \equiv 3x - 2 + \frac{(12 - 4x)}{x^2 - 4}$$

NB: Check Illustration 2 above

Aside

$$\frac{(12 - 4x)}{x^2 - 4} \equiv \frac{12 - 4x}{(x - 2)(x + 2)}$$

Let $\frac{12-4x}{(x-2)(x+2)} \equiv \frac{A}{x-2} + \frac{B}{x+2}$

$$\equiv \frac{A(x + 2) + B(x - 2)}{(x - 2)(x + 2)}$$

$\Rightarrow A(x + 2) + B(x - 2) = 12 - 4x$

When $x = 2$: $4A = 4$

$\therefore A = 1$

When $x = -2$: $-4B = 20$

$\therefore B = -5$

Now

$$\frac{12 - 4x}{(x - 2)(x + 2)} \equiv \frac{1}{x - 2} - \frac{5}{x + 2}$$

$$\therefore \frac{3x^3 - 2x^2 - 16x + 20}{x^2 - 4} \equiv 3x - 2 + \frac{1}{x - 2} - \frac{5}{x + 2}$$

Follow Up Questions

Question 1

Resolve $\frac{x+11}{x^2-2x-15}$ into partial fractions

Answer: $\frac{2}{x-5} - \frac{1}{x+3}$

Question 2

Convert $\frac{2x+5}{(x-2)(x+1)}$ into partial fractions

Answer: $\frac{3}{x-2}-\frac{1}{x+1}$

Question 3

Express $\frac{10-x}{x^2+x-2}$ in partial fractions

Answer: $\frac{3}{x-1}-\frac{4}{x+2}$

Question 4

Resolve $\frac{1}{x^2-4}$ into partial fractions

Answer: $\frac{1}{4(x-2)}-\frac{1}{4(x+2)}$

Question 5

Convert $\frac{2x+3}{x^2-9}$ into the sum of two partial fractions

Answer: $\frac{1}{2(x+3)}+\frac{3}{2(x-3)}$

Question 6

Express $\frac{2-x}{x^2+5x}$ in partial fractions

Answer: $\frac{2}{5x}-\frac{7}{5(x+5)}$

Question 7

Resolve $\frac{x^4+x^3+x^2+1}{x^2+x-2}$ into partial fractions

Answer: $x^2+3+\frac{1}{3(x+2)}-\frac{1}{2(-1)}$

Question 8

Convert $\frac{x^2-1}{(x+4)(x-4)}$ into partial fractions

Answer: $1-\frac{15}{8(x+4)}+\frac{15}{8(x-4)}$

Question 9

Express $\frac{x^2+x-1}{x(x^2-1)}$ in partial fractions Answer: $\frac{1}{x}-\frac{1}{2(x+1)}+\frac{1}{2(x-1)}$

Question 10

Express $\frac{5x-7}{(x+1)(x-2)}$ in partial fractions Answer: $\frac{4}{x+1}-\frac{1}{x-2}$

Question 11

Express $\frac{3x+11}{(x+2)(x-3)}$ in partial fractions Answer: $\frac{4}{x-3}-\frac{1}{x+2}$

Question12

Resolve $\frac{x^2}{x^2-1}$ into partial fractions Answer: $1+\frac{1}{2(x-1)}-\frac{1}{2(x+1)}$

Question 13

Express $\frac{2x^2+x-4}{x^2-3x+2}$ in partial fractions Answer: $2+\frac{1}{x-1}+\frac{6}{x-2}$

Question 14

Resolve $\frac{4x^2-47x+141}{x^2-13x+40}$ into partial fractions Answer: $4-\frac{2}{x-5}+\frac{7}{x-8}$

Question 15

Express $\frac{5x^2-77}{x^2-2x-15}$ in partial fractions Answer: $5+\frac{4}{x+3}+\frac{6}{x-5}$

Question 16

Resolve $\frac{4x^2+3x-16}{(x-2)(x+1)(2x-3)}$ into partial fractions Answer: $\frac{2}{x-2}-\frac{1}{x+1}+\frac{2}{2x-3}$

B. Repeated Linear Factors

- The denominators contain repeated linear factors
- These are of the form

$$\frac{f(x)}{(x+a)^n}$$

- This type of partial fractions can be resolved as

$$\frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{C}{(x+a)^3} + \cdots + \frac{D}{(x+a)^n}$$

Worked Problems

Example 1

Express $\frac{4}{x(x+2)^2}$ in partial fractions

Suggested Solution

$$\frac{4}{x(x+2)^2}$$

Let $\frac{4}{(x+2)^2} \equiv \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$

$$\equiv \frac{A(x+2)^2 + Bx(x+2) + Cx}{x(x+2)^2}$$

$\Rightarrow A(x+2)^2 + Bx(x+2) + Cx = 4$

When $x = -2$: $-2C = 4$

$\therefore C = -2$

When $x = 0$: $4A = 4$

$\therefore A = 1$

Comparing coefficients of x^2: $A + B = 0$

$$\therefore B = -1$$

Now

$$\frac{4}{(x+2)^2} \equiv \frac{1}{x} - \frac{1}{x+2} - \frac{2}{(x+2)^2}$$

Example 2

Convert $\frac{x+6}{x^2(x-3)}$ into the sum of three partial fractions

Suggested Solution

$$\frac{x+6}{x^2(x-3)}$$

Let $\frac{x+6}{x^2(x-3)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3}$

$$\equiv \frac{Ax(x-3) + B(x-3) + Cx^2}{x^2(x-3)}$$

$\Rightarrow Ax(x-3) + B(x-3) + Cx^2 = x + 6$

When $x = 0$: $-3B = 6$

$\therefore B = -2$

When $x = 3$: $9C = 9$

$\therefore C = 1$

Comparing coefficients of x^2: $A + C = 0$

$\therefore A = -1$

Now

$$\frac{x+6}{x^2(x-3)} \equiv \frac{1}{x-3} + -\frac{1}{x} - \frac{2}{x^2}$$

Example 3

Resolve $\frac{3x^3-2x^2-16x+20}{x^2(x-4)}$ into partial fractions

Suggested Solution

$\frac{3x^3-2x^2-16x+20}{x^2(x-4)}$ is an improper fractions

Aside

$$\frac{3x^3-2x^2-16x+20}{x^2(x-4)} \equiv \frac{3x^3-2x^2-16x+20}{x^3-4x^2}$$

$$\begin{array}{r|l} & \quad 3 \\ \hline x^3-4x^2 & 3x^3-2x^2-16x+20 \\ & -\underline{\left(3x^3-12x^2\right)} \\ & \qquad 10x^2-16x+20 \end{array}$$

$$\therefore \frac{3x^3-2x^2-16x+20}{x^2(x-4)} \equiv 3+\frac{10x^2-16x+20}{x^2(x-4)} \quad \{\text{It is now proper}\}$$

Let $\frac{10x^2-16x+20}{x^2(x-4)} \equiv \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-4}$

$$\equiv \frac{Ax(x-4)+B(x-4)+Cx^2}{x^2(x-4)}$$

$\Rightarrow Ax(x-4)+B(x-4)+Cx^2 = 10x^2-16x+20$

When $x=0$: $-4B=20$ When $x=4$: $16C=116$

$\therefore B = -5$ $\qquad\qquad$ $\therefore C = \frac{29}{4}$

Comparing coefficients of x^2: $A + C = 10$

$$A = 10 - \frac{29}{4}$$

$$\therefore A = \frac{11}{4}$$

Now

$$\frac{10x^2 - 16x + 20}{x^2(x-4)} \equiv \frac{11}{4x} - \frac{5}{x^2} + \frac{29}{4(x-4)}$$

$$\therefore \frac{3x^3 - 2x^2 - 16x + 20}{x^2(x-4)} \equiv 3 + \frac{11}{4x} - \frac{5}{x^2} + \frac{29}{4(x-4)}$$

Follow Up Questions

Question 1

Resolve $\frac{1}{x^3 - x^2}$ into partial fractions

Answer: $\frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2}$

Question 2

Convert $\frac{x^2+1}{x(x-1)^3}$ into partial fractions

Answer: $\frac{1}{x-1} - \frac{2}{(x-1)^3} - \frac{1}{x}$

Question 3

Express $\frac{3x^2+6x+5}{(x+2)^2(x-3)}$ in partial fractions

Answer: $\frac{1}{x+2} - \frac{1}{(x+2)^2} + \frac{2}{x-3}$

Question 4

Resolve $\frac{x+7}{x^2(x+2)}$ into partial fractions Answer: $-\frac{5}{4x}+\frac{7}{2x^2}+\frac{5}{4(x+2)}$

Question 5

Convert $\frac{2x+3}{(x-2)^2}$ into the sum of two partial fractions Answer: $\frac{2}{x-2}+\frac{7}{(x-2)^2}$

Question 6

Express $\frac{5x^2-2x-19}{(x+3)(x-1)^2}$ in partial fractions Answer: $\frac{2}{x+3}-\frac{3}{x-1}-\frac{4}{(x-1)^2}$

Question 7

Resolve $\frac{3x^2+16x+15}{(x+3)^3}$ into partial fractions Answer: $\frac{3}{x+3}-\frac{2}{(x+3)^2}-\frac{6}{(x+3)^3}$

Question 8

Convert $\frac{x^2+7x+3}{x^2(x+3)}$ into partial fractions Answer: $\frac{1}{x^2}+\frac{2}{x}-\frac{1}{x+3}$

Question 9

Express $\frac{-x^2+21x+18}{(x-5)(x+2)^2}$ in partial fractions Answer: $\frac{2}{x-5}-\frac{3}{x+2}+\frac{4}{(x+2)^2}$

Question 10

Express $\frac{5x^2-30x+44}{(x-2)^3}$ in partial fractions Answer: $\frac{5}{x-2}-\frac{10}{(x-2)^2}+\frac{4}{(x-2)^3}$

C. Quadratic Factors

- The denominators contain quadratic factors which cannot be factorised

- These are of the form

$$\frac{f(x)}{(ax^2+bx+c)(x+d)}$$

- This type of partial fractions can be resolved as

$$\frac{Ax+B}{(ax^2+bx+c)}+\frac{C}{(x+d)}$$

Worked Problems

Example 1

Express $\frac{2x+4}{(x-1)(x^2+2)}$ in partial fractions

Suggested Solution

$$\frac{2x+4}{(x-1)(x^2+2)}$$

Let $\frac{2x+}{(x-1)(x^2+2)} \equiv \frac{A}{x-1}+\frac{Bx+C}{(x^2+2)}$

$$\equiv \frac{A(x^2+2)+(Bx+C)(x-1)}{(x-1)(x^2+2)}$$

$\Rightarrow A(x^2+2)+(Bx+C)(x-1)=2x+4$

When $x=1$: $3A=6$

$\therefore A=2$

When $x=0$: $2A-C=4$

$2(2)-4=C$

$\therefore C=0$

Comparing coefficients of x^2: $A + B = 0$

$$2 + B = 0$$

$$\therefore B = -2$$

Now

$$\frac{2x +}{(x-1)(x^2+2)} \equiv \frac{2}{x-1} - \frac{2x}{x^2+2}$$

Example 2

Convert $\frac{x^2-x-13}{(x^2+7)(x-2)}$ into partial fractions

Suggested Solution

$$\frac{x^2-x-13}{(x^2+7)(x-2)}$$

Let $\frac{x^2-x-13}{(x^2+7)(x-2)} \equiv \frac{Ax+B}{x^2+7} + \frac{C}{x-2}$

$$\equiv \frac{(Ax+B)(x-2)+C(x^2+7)}{(x^2+7)(x-2)}$$

$\Rightarrow (Ax+B)(x-2)+C(x^2+7) = x^2-x-13$

When $x = 2$: $11C = -11$

$$\therefore C = -1$$

When $x = 0$: $-2B + 7C = -13$

$$-2B - 7 = -13$$

$$\therefore B = 3$$

Comparing coefficients of x^2: $A + C = 1$

$$A - 1 = 1$$

$$\therefore A = 2$$

Now

$$\frac{x^2 - x - 13}{(x^2+7)(x-2)} \equiv \frac{2x+3}{x^2+7} - \frac{1}{x-2}$$

Example 3

Resolve $\frac{3x^3-2x^2-16x+20}{(x^2+1)(x+2)}$ into partial fractions

<u>Suggested Solution</u>

$\frac{3x^3-2x^2-16x+20}{(x^2+1)(x+2)}$ is an improper fractions

<u>Aside</u>

$$\frac{3x^3 - 2x^2 - 16x + 20}{(x^2+1)(x+2)} \equiv \frac{3x^3 - 2x^2 - 16x + 20}{x^3 + 2x^2 + x + 2}$$

$$\begin{array}{r|l}
 & \quad 3 \\
\hline
x^3 + 2x^2 + x + 2 & 3x^3 - 2x^2 - 16x + 20 \\
 & -\underline{\left(3x^3 + 6x^2 + 3x \quad + 6\right)} \\
 & \qquad -8x^2 - 19x + 14
\end{array}$$

$$\therefore \frac{3x^3 - 2x^2 - 16x + 20}{(x^2+1)(x+2)} \equiv 3 - \frac{8x^2 + 19x - 14}{(x^2+1)(x+2)} \quad \{\text{ It is now proper}\}$$

Let $\frac{8x^2+19x-14}{(x^2+1)(x+2)} \equiv \frac{Ax+B}{x^2+1} + \frac{C}{x+2}$

$$\equiv \frac{(Ax+B)(x+2)+C(x^2+1)}{(x^2+1)(x+2)}$$

$\Rightarrow (Ax+B)(x+2)+C(x^2+1) = 8x^2+19x-14$

When $x=-2$: $5C=-20$

$\therefore C=-4$

When $x=0$: $2B+C=-14$

$2B-4=-14$

$\therefore B=-5$

Comparing coefficients of x^2: $A+C=8$

$A=8-(-4)$

$\therefore A=12$

Now

$$\frac{8x^2+19x-14}{(x^2+1)(x+2)} \equiv \frac{12x-5}{x^2+1} - \frac{4}{x+2}$$

$$\therefore \frac{3x^3-2x^2-16x+20}{(x^2+1)(x+2)} \equiv 3 - \frac{12x-5}{x^2+1} + \frac{4}{x+2}$$

Follow Up Questions

Question 1

Resolve $\frac{7x^2+5x+13}{(x^2+2)(x+1)}$ into partial fractions

Answer: $\frac{2x+3}{x^2+2} + \frac{5}{x+1}$

Question 2

Convert $\frac{3+6x+4x^2-2x^3}{x^2(x^2+3)}$ into partial fractions

Answer: $\frac{2}{x} - \frac{1}{x^2} + \frac{3-4x}{x^2+3}$

Question 3

Express $\frac{x^2-x-13}{(x^2+7)(x-2)}$ in partial fractions

Answer: $\frac{2x+3}{x^2+7}-\frac{1}{x-2}$

Question 4

Resolve $\frac{6x-5}{(x-4)(x^2+3)}$ into partial fractions

Answer: $\frac{1}{x-4}+\frac{2-x}{x^2+3}$

Question 5

Convert $\frac{15+5x+5x^2-4x^3}{x^2(x^2+5)}$ to partial fractions

Answer: $\frac{1}{x}+\frac{3}{x^2}+\frac{2-5x}{x^2+5}$

Question 6

Express $\frac{x^3+4x^2+20x-7}{(x-1)^2(x^2+8)}$ in partial fractions

Answer: $\frac{3}{x-1}+\frac{2}{(x-1)^2}+\frac{1-2x}{x^2+8}$

Question 7

Resolve $\frac{1}{x^3+x}$ into partial fractions

Answer: $\frac{1}{x}-\frac{x}{x^2+1}$

Question 8

Convert $\frac{3x^3+7x-4}{(x^2+2)^2}$ into partial fractions

Answer: $\frac{3x}{x^2+2}+\frac{x-4}{(x^2+2)^2}$

Question 9

Express $\frac{x^2+15}{(x^2+3)(x+3)^2}$ in partial fractions

Answer: $\frac{1}{2(x+3)}+\frac{2}{(x+3)^2}+\frac{1-x}{2(x^2+3)}$

Question 10

Express $\frac{4x^2+16x+7}{(x^2+4)^2}$ in partial fractions

Answer: $\frac{4x}{(x^2+4)}+\frac{7}{(x^2+4)^2}$

PAST EXAMINATION QUESTION

UCLES PAPER 1 JUNE 1998

a) Express $\frac{1}{(x+3)(4-x)}$ in partial fractions. **[2]**

Hence find

$$\int_0^2 \frac{1}{(x+3)(4-x)}\,dx$$ [4]

ZIMSEC JUNE 2004 PAPER 1

Express $\frac{6-x}{(2x+1)(x^2+3)}$ in the form $\frac{A}{2x+3}+\frac{Bx+C}{x^2+3}$ where A, B and C are numerical constants to be found. **[3]**

Hence show that

$$\int_0^1 \frac{6-x}{(2x+1)(x^2+3)}\,dx = In\left(\frac{3\sqrt{2}}{2}\right)$$ [4]

ZIMSEC NOVEMBER 2004 PAPER 1

Express $\frac{1}{x(1000-x)}$ in partial fractions and hence show that

$$\int \frac{1}{x(1000-x)}\,dx = \frac{1}{1000} In\left(\frac{x}{1000-x}\right) + c.$$ [6]

ZIMSEC JUNE 2006 PAPER 1

Express $\frac{5}{(x+1)(x^2+4)}$ in partial fractions. **[4]**

ZIMSEC JUNE 2008 PAPER 1

a) Express $\frac{11x-1}{(x+1)^2(x-3)}$ in the form $\frac{A}{(x+1)^2}+\frac{B}{x+1}+\frac{C}{x-3}$. **[4]**

b) Hence evaluate

$$\int \frac{11x-1}{(x+1)^2(x-3)}\,dx$$ [3]

ZIMSEC NOVEMBER 2008 PAPER 1

Express $\frac{1+x}{(2+x)(3x+5)}$ in partial fractions. **[2]**

(i) By putting $x=\sqrt{3}$, or otherwise, obtain an expression for

$$\frac{1+\sqrt{3}}{(2+\sqrt{3})(3\sqrt{3}+5)}$$

in the form $a+b\sqrt{3}$, where a and b are integers. **[3]**

(ii) Show that

$$\int_{-1}^{1}\frac{1+x}{(2+x)(3x+5)}\,dx = In3-\frac{4}{3}In2.$$ [4]

ZIMSEC JUNE 2009 PAPER 1

a) Express $\frac{7x^2+2x+20}{(x^2+4)(x+1)}$ in partial fractions. **[4]**

b) Evaluate exactly

$$\int_3^5 \frac{7x^2 + 2x + 20}{(x^2 + 4)(x + 1)} dx$$ **[5]**

c) Find the coefficients of x^5 and x^6 in the expansion of $\frac{7x^2+2x+20}{(x^2+4)(x+1)}$. **[5]**

ZIMSEC NOVEMBER 2009 PAPER 1

Express in partial fractions.

$$\frac{5x^2+7x+9}{(x+2)^2(3-x)}$$ **[5]**

ZIMSEC JUNE 2010 PAPER 1

Express $\frac{3x+8}{(2x+1)(x^2+3)}$ in partial fractions. **[4]**

ZIMSEC NOVEMBER 2010 PAPER 1

Show that $2x^3 - x^2 + 8x - 4 = (2x - 1)(x^2 + 4)$. **[2]**

i) Express $\frac{x^2+2x+20}{2x^3-x^2+8x-4}$ in the partial fractions **[4]**

ii) Hence evaluate

$$\int_1^3 \frac{x^2 + 2x + 20}{2x^3 - x^2 + 8x - 4} dx$$

giving your answer correct to three significant figures. **[4]**

ZIMSEC NOVEMBER 2011 PAPER 1

a) Express the function $\frac{3}{x^2(x+3)}$ in partial fractions. **[3]**

b) Hence evaluate the integral of

$$\int_2^4 \frac{3}{x^2(x+3)}\,dx,$$

leaving your answer in exact form. **[4]**

ZIMSEC JUNE 2012 PAPER 1

Express $\frac{x^4}{x^4-1}$ in partial fractions. **[5]**

ZIMSEC JUNE 2013 PAPER 1

(i) Express $\frac{4x+6}{(x+2)(x+1)(x+3)}$ in partial fractions. **[3]**

(ii) By using a series expansion up to and including the term in x^2 show that

$\frac{4x+6}{(x+2)(x+1)(x+3)}$ can be reduced to $1-\frac{7}{6}x+\frac{41}{36}x^2$. **[4]**

ZIMSEC JUNE 2013 PAPER 2

Show that

$$\int_1^2 \frac{x^2+6x+7}{(x+2)(x+3)}\,dx = In\left(\frac{75e}{64}\right)$$ [5]

ZIMSEC NOVEMBER 2014 PAPER 1

Express $\frac{2x^3-17x-1}{(x-2)(x^2+5)}$ in partial fractions. **[6]**

ZIMSEC NOVEMBER 2015 PAPER 1

Express $\frac{5x^2-5x+11}{(x-2)(x^2+3)}$ in partial fractions. **[5]**

ZIMSEC NOVEMBER 2016 PAPER 1

(i) Express $\frac{x-3}{x^2-1}$ in partial fractions. **[2]**

(ii) Hence from (i) find

$$\int_{-1}^{1} \frac{x-3}{x^2-1}\,dx,$$

giving your answer in terms of a single logarithm. **[4]**

ZIMSEC JUNE 2017 PAPER 1

The function $(x) = \frac{x^3+2x^2+x+2}{x^2+x}$.

(a) Express $f(x)$ in the form $P(x) + \frac{A}{x} + \frac{B}{x+1}$, where A and B are constants and $P(x)$ is a function of x. **[4]**

(b) Hence find the exact value of

$$\int_{1}^{2} f(x)dx$$ [**4**]

ZIMSEC NOVEMBER 2017 PAPER 1

Express $\frac{3x+8}{(2x+1)(x^2+3)}$ in partial fractions. **[5]**

ZIMSEC NOVEMBER 2017 PAPER 2

Find the exact value of

$$\int_1^2 \frac{3}{x^2(3-x)}\,dx$$ [**8**]

ZIMSEC JUNE 2018 PAPER 1

Express $\frac{x^3-x^2-x-4}{x(x^2+2)}$ in partial fractions. **[6]**

ZIMSEC NOVEMBER 2018 PAPER 1

The function $(x) = \frac{5x^2-5x+11}{(x-2)(x^2+3)}$.

(c) Express $f(x)$ in partial fractions. **[5]**

(d) Hence or otherwise evaluate

$$\int_0^2 f(x)dx,$$

giving your answer as an exact single term. **[5]**

ZIMSEC NOVEMBER 2018 PAPER 2

a) Express $\frac{2x^2-7}{(x-3)(2x+5)}$ in the form $A + \frac{B}{x-3} + \frac{C}{2x+5}$. **[5]**

b) Hence evaluate

$$\int_4^5 \frac{2x^2-7}{(x-3)(2x+5)}\,dx$$

correct to 3 decimal places. **[4]**

ZIMSEC JUNE 2019 PAPER 1

The function $f(x) = \frac{x^3-x-2}{(x-1)(x^2+1)}$

(a) Express $f(x)$ in the form $A + \frac{B}{x-1} + \frac{Cx+D}{x^2+1}$ where, A, B, C and D are constants. **[5]**

(b) Hence evaluate

$$\int_2^3 f(x)\ dx$$ [**4**]

ZIMSEC NOVEMBER 2019 PAPER 1

(a) Express $f(x) = \frac{4x+5}{(x+1)(2x+1)^2}$ in partial fractions. **[5]**

(b) Hence find the exact value of

$$\int_0^2 f(x)\ dx$$ [**4**]

ASANTE SANA

*******THERE IS A LIGHT AT THE END OF EVERY TUNNEL *******

CONSTRUCTIVE COMMENTS ON THE FORM OF THE PRESENTATION, INCLUDING ANY OMISSIONS OR ERRORS, ARE WELCOME.

ENJOY

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