Plane Trigonometry

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SYLLABUS (6042) REQUIREMENTS

- distinguish between radians and degrees
- convert degrees to radians and vice versa
- calculate arc length and sector area
- solve problems involving lengths of arcs, areas of sectors and segments
- use small angle approximation for $\sin x$, $\cos x$ and $\tan x$

PLANE TRIGONOMETRY

- The word **trigonometry** comes from the Greek words *trigon* and *metron*, which mean triangle and measure.
- The study of trigonometry involves triangle measurement.

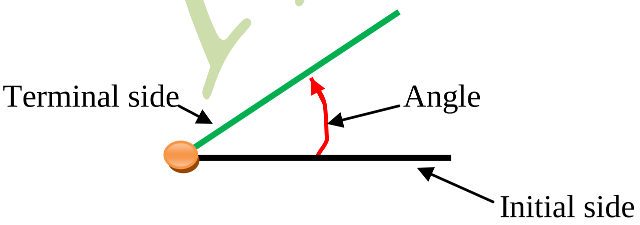
RADIANS AND DEGREES

Angles

Definition

An angle is a measure of rotation usually measured in degrees from the initial side to the terminal side.

- An angle is formed from two rays joined end to end.



- An angle in standard position is an angle where the initial side is the $x-axis$.
- Angles can be measured in units of either degrees or radians.

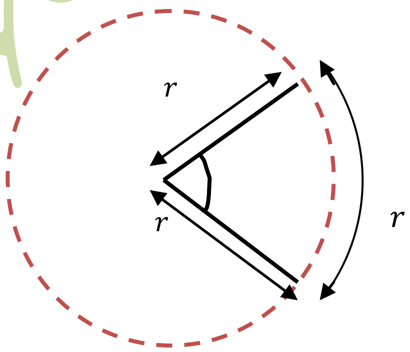
- To go from the standard position back to the original standard position, the angle would have to be 360°.
- When and angle is measured counter-clockwise/ anti-clockwise is a positive angle measurement
- When and angle is measured clockwise is a negative angle measurement
- When we measure angles we can think of them in terms of pieces of a circle.
- We have two units for measuring angles which are **Degrees** and **Radians**.
- The symbol for degree is ° and usually no symbol is used to denote radians.

Degrees

One **Degree** (1°) is a rotation of 1/360 of a complete revolution about the vertex.
We can think of this as going all the way around the circle. There are 360 degrees in one full rotation.

Radians

One **Radian** is the measure of a central angle θ that intercepts an arc s equal in length to the radius r of the circle or simply a central angle with arc length equal to 1 radius is called 1 radian



The arc length for a full circle is the same as its circumference, $2\pi r$.
Also remember that the arc length = $r\theta$.

$$\therefore 2\pi r = r\theta$$

$$\Rightarrow \theta = 2\pi$$

Since the radian is measured in terms of one r on the arc of a circle and the complete circumference of the circle is $2\pi r$ then there are 2π radians in one full rotation.

NOTE: A complete revolution is defined as $360°$ or $2\pi\ radians$ and half a revolution has $180°$ or $\pi\ radians$

This allows us to give equivalences between degrees and radians.

$$\Rightarrow 1° = \frac{2\pi}{360}\ radians$$

$$\Rightarrow 1radian = \frac{360}{2\pi}\ degrees$$

Converting Radians to Degrees

Multiply by $\frac{360}{2\pi}$ (getting rid of radians)

Example

Convert:

(i) $\frac{3}{4}\pi$ to degrees

(ii) 1.2 $radians$ to degrees

Solution

(i) $\frac{3}{4}\pi$ to degrees

$$\frac{3\pi}{4} \times \frac{360}{2\pi} = \frac{3}{4} \times \frac{360}{2} = \frac{3}{4} \times 180° = 3 \times 45° = 135°$$

(ii) 1.2 $radians$ to degrees

$$1.2 \times \frac{360}{2\pi} = 1.2 \times 57.29577951° = 68.75493542° \approx 69°$$

Converting Degrees to Radians

Multiply by $\frac{2\pi}{360}$ (getting rid of ° notation)

Example

Convert:

(i) 270° to radians

(ii) 675 ° to radians

Solution

(i) 270° to radians

$$270° \times \frac{2\pi}{360} = 3 \times \frac{2\pi}{4} = 3 \times \frac{\pi}{2} = \frac{3\pi}{2} radians$$

(ii) 675 ° to radians

$$675° \times \frac{2\pi}{360} = 15 \times \frac{2\pi}{8} = 15 \times \frac{\pi}{4} = \frac{15\pi}{4} radians$$

NOTE: Your calculator should be able to work with angles measured in both radians and degrees. Usually the MODE button allows you to select the appropriate measure. When calculations involve calculus you should always work with radians and not degrees.

Follow up Exercise

1. Convert each of the following angles given in degrees, to radians. Give your answers correct to 2 *decimal places*.

   a) 5°, b) 63°, c) 193°.

2. Convert each of the following angles given in radians, to degrees. Give your answers correct to 1 *decimal place*.

a) $2.1\ radians$, b) $8.93\ radians$, c) $1.06\ radians$.

3. Convert each of the following angles given in radians, to degrees. Do not use a calculator.

   a) $\frac{6\pi}{5}$, b) $\frac{\pi}{8}$.

4. Convert the following angles given in degrees, to radians. Do not use a calculator and give your answers as multiples of π.

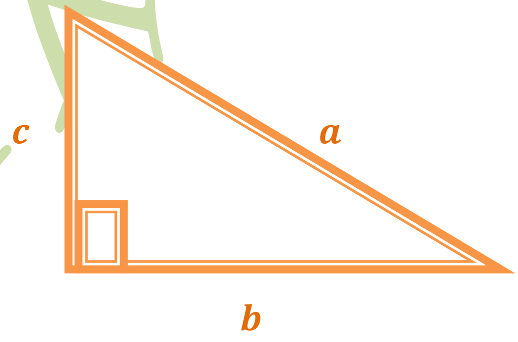
   a) $15°$, b) $120°$, c) $-90°$.

Answers

1. a) $0.09\ radians$, b) $1.10\ radians$, c) $3.37\ radians$.
2. a) $120.3°$, b) $511.7°$, c) $60.7°$.
3. a) $216°$, b) $22.5°$.
4. a) $\frac{\pi}{12}$, b) $\frac{2\pi}{3}$, c) $-\frac{\pi}{4}$.

RIGHT ANGLED TRIANGLES

In a right triangle, the side opposite the right angle is the longest side. We call this side the hypotenuse of the triangle. The two sides that form the right angle are called legs.



NOTE

(i) The side "c" is always the side opposite the right angle. Side "a" is called the hypotenuse.

(ii) The sides adjacent to the right angle are named "b" and "c".

(iii)There is no choice when selecting a side to be named "a", but the sides you choose to call "b" and "c" are interchangeable.

Pythagoras' Theorem

- Pythagoras' theorem is used in determining the distance between two points in both two and three dimensional space.
- The Pythagoras' Theorem is a formula that gives a relationship between the sides of a right triangle The Pythagorean Theorem only applies to RIGHT triangles. A RIGHT triangle is a triangle with a 90 degree angle. The 90 degree angle in a right triangle is often depicted with a square.
- Legs – the shorter two sides opposite the acute angles.

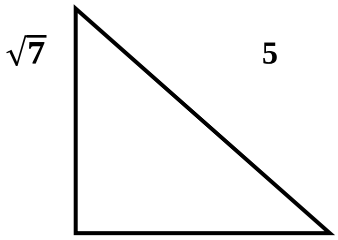
Pythagoras' Theorem: $hypotenuse^2 = leg^{\,2} + leg^{\,2}$

$$a^2 = b^2 + c^{\,2}$$

- Triples of integers such as $(3, 4, 5)$ and $(5, 12, 13)$ which occur as the side lengths of right-angled triangles are of great interest in both geometry and number theory and they are called Pythagorean triples.

Example

Calculate the missing side x.



Solution

$a^2 = b^2 + c^{\,2}$

$\Longrightarrow (5)^2 = x^2 + (\sqrt{7})^{\,2}$

$\Longrightarrow 25 = x^2 + 7$

$\Longrightarrow x^2 = 25 - 7$

$\Longrightarrow x^2 = 18$

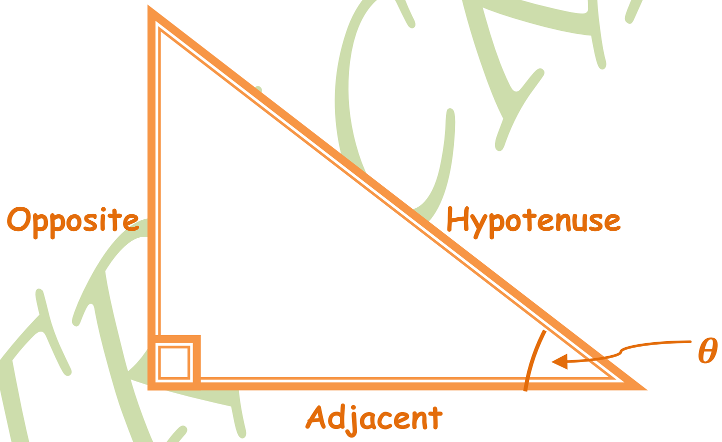
$\Longrightarrow x = \sqrt{18}$

$\Longrightarrow x = \sqrt{9 \times 2}$

$\therefore x = 3\sqrt{2}.$

Trigonometric Ratios

- The trig ratios are just the ratios formed by the lengths of the sides of a right triangle.
- There are 6 trig ratios namely, $\cos x$, $\tan x$, $\sin x$, $\sec x$, $cosec\, x$ and $\cot x$, though we will only be working with the first 3



- The side labelled **hypotenuse** is always opposite the right angle of the right triangle. The Hypotenuse is the longest side of a right triangle; the side opposite (across) from the right angle.
- The names of the other two sides of the right triangle are determined by the angle θ that is being discussed.
- The angle θ is formed by the hypotenuse of the right triangle and the side of the right triangle that is called adjacent.

- The **adjacent** side will always make up part of the angle that is being discussed and not be the hypotenuse. Adjacent means to be next to or touching
- The side of the right triangle that does not form part of angle θ is called the **opposite** side. The opposite side will never form part of the angle being discussed. **Opposite** means to be across from.
- The three ratios are:

$$cos\theta = \frac{Adjacent}{Hypotenuse} \ (CHA)$$

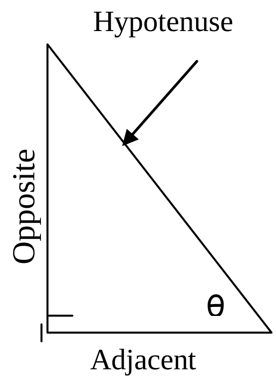
$$sin\theta = \frac{Opposite}{Hypotenuse} \ (SHO)$$

$$tan\theta = \frac{Opposite}{Adjacent} \ (TAO)$$

sin = shortened form of sine function; cos = shortened form of cosine function; tan = shortened form of tangent function and θ = the angle value.

MNEMONIC (to remember more easily): $CHASHOTAO$

SUMMARY

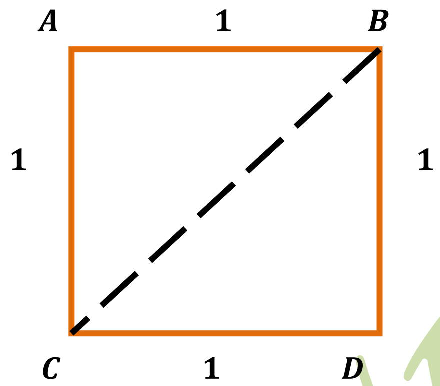
	Name	Symbol	Definition	
Trigonometric Ratios	$sine\ \theta$	$sin\ \theta$	$\frac{\text{opposite side}}{\text{hypotenuse}}$	
	$cosine\ \theta$	$cos\ \theta$	$\frac{\text{adjacent side}}{\text{hypotenuse}}$	
	$tangent\ \theta$	$tan\ \theta$	$\frac{\text{opposite side}}{\text{adjacent side}}$	

NOTE: The hypotenuse is always the same regardless which acute angle you label as θ. The adjacent and opposite sides will switch depending which acute angle you label as θ.

SPECIAL TRIANGLES

The cosine, sine and tangent of some basic angles is derived by considering some special right triangles.

Let's consider a square with sides of length 1, and its diagonal as shown below.



The length of the diagonal BC is calculated using the Pythagorean theorem as follows:

$BC^2 = 1^2 + 1^2$

$BC^2 = 2$

$\therefore BC = \sqrt{2}$

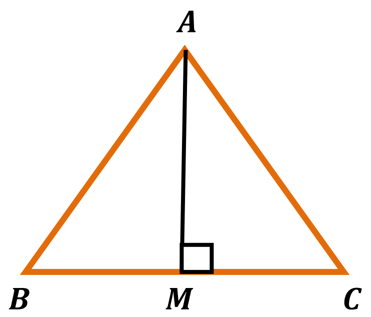
NB: $\boldsymbol{A\hat{B}C = A\hat{C}B = D\hat{C}B = D\hat{B}C = 45°}$

Now:

$Tan\theta = \frac{Opposite}{Adjacent}$; $\quad Sin\theta = \frac{Opposite}{Hypotenuse}$; $\quad Cos\theta = \frac{Adjacent}{Hypotenuse}$

$Tan\ 45 = \frac{1}{1} = 1 \quad Sin\ 45 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \quad Cos\ 45 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Now let's consider for 60° and 30°. Lets draw an equilateral triangle with sides of length 2.



It is clear that all angles are equal and add to $180°$ or $\pi\ radians$. Thus each angle is a 60 angle or has a measure of $\frac{\pi}{3} radians$.

Additionally, we know that the line which bisects the upper angle in this (isosceles) triangle also bisects the base.

NB: $\hat{B}M = A\hat{C}M = 60°$; $C\hat{A}M = B\hat{A}M = 30°$ and $A\hat{M}B = A\hat{M}C = 90°$. Also $MC = MB = 1\ unit$ and $AB = AC = 2units.$

Using the Pythagorean Theorem we can find the length of the bisector as follows:

$AC^2 = AM^2 + MC^2$

$2^2 = AM^2 + 1^2$

$AM^2 = 4 - 1$

$AM^2 = 3$

$\therefore AM = \sqrt{3}$

Now:

$Tan\theta = \frac{Opposite}{Adjacent}$; $Sin\theta = \frac{Opposite}{Hypotenuse}$; $Cos\theta = \frac{Adjacent}{Hypotenuse}$

$Tan\ 30 = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $Sin\ 30 = \frac{1}{2}$ $Cos\ 30 = \frac{\sqrt{3}}{2}$

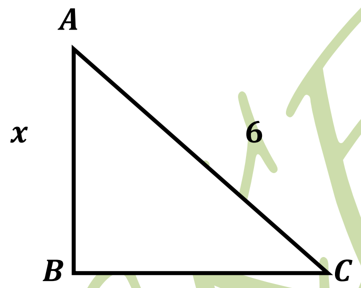
$Tan\ 60 = \frac{\sqrt{3}}{1} = \sqrt{3}$ $Sin\ 60 = \frac{\sqrt{3}}{2}$ $Cos\ 60 = \frac{1}{2}$

We can summarise the above ratios in a table.

RATIO	30°	45°	60°
$Sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$Cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$Tan\theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Example

Calculate the exact value of x if $A\hat{C}B = 45°$.



Solution

$Sin\theta = \frac{Opposite}{Hypotenuse}$

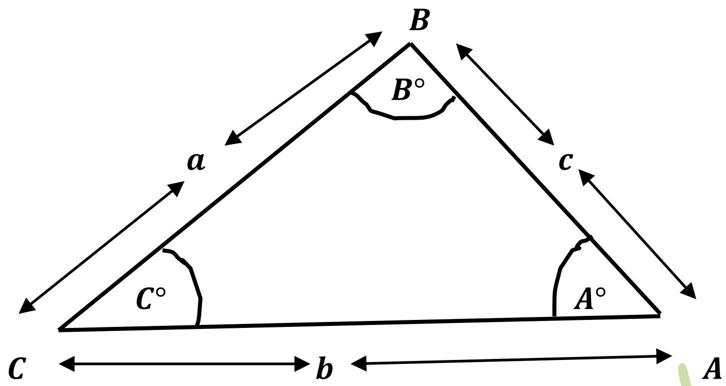
$\Rightarrow Sin45 = \frac{x}{6}$

$\Rightarrow x = 6sin45$

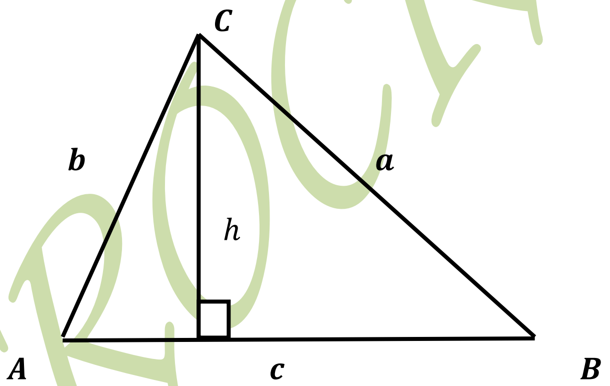
$\Rightarrow x = 6\left(\frac{\sqrt{2}}{2}\right)$

$\therefore x = 3\sqrt{2}.$

TRIANGLE LABELLING



THE SINE RULE



From the triangle ABC above, we can find h in two different ways:

$$Sin\theta = \frac{Opposite}{Hypotenuse}$$

$$\Longrightarrow Sin(A\hat{B}C) = \frac{h}{a}$$

$$\therefore h = aSinB$$

Also,

$$Sin(B\hat{A}C) = \frac{h}{b}$$

$$\therefore h = bSinA$$

$$\Longrightarrow aSinB = bSinA \qquad \text{(i)}$$

Now for sides we divide both sides of equation (i) by $SinB$ first and then we divide both sides by $SinA$ to get:

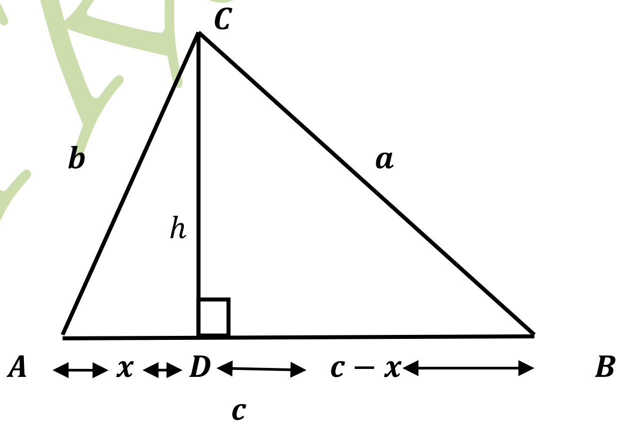
$$\frac{a}{SinA} = \frac{b}{SinB}$$

Now for angles we divide both sides of equation (i) by a first and then we divide both sides by b to get:

$$\frac{Sin\ A}{a} = \frac{Sin\ B}{b}$$

NB: $Sin\ \theta\ \ \{\boldsymbol{acute}\} = Sin\ (180 - \theta)\ \ \{\boldsymbol{obtuse}\}$

THE COSINE RULE



Using the Pythagoras Theorem for triangle ACD:

$$b^2 = h^2 + x^2$$

$$\therefore h^2 = b^2 - x^2 \qquad \text{(i)}$$

Using the Pythagoras Theorem for triangle BCD:

$$a^2 = h^2 + DB^2$$

$$a^2 = h^2 + (c-x)^2$$

$$\therefore h^2 = a^2 - (c-x)^2 \qquad \text{(ii)}$$

Substituting (i) into (ii):

$$h^2 = a^2 - (c-x)^2 \qquad \text{(ii)}$$

$$a^2 - (c-x)^2 = b^2 - x^2$$

$$a^2 - (c^2 - 2cx + x^2) = b^2 - x^2$$

$$a^2 - c^2 + 2cx - x^2 = b^2 - x^2$$

$$\Longrightarrow a^2 - c^2 + 2cx = b^2$$

$$\therefore a^2 = b^2 + c^2 - 2cx \qquad \text{(iii)}$$

Now using triangle ACD again:

$$Cos\theta = \frac{Adjacent}{Hypotenuse}$$

$$\Longrightarrow CosA = \frac{x}{b}$$

$$\therefore x = bCosA \qquad \text{(iv)}$$

Substituting (iv) into (iii):

$$a^2 = b^2 + c^2 - 2cx \qquad \text{(iii)}$$

$$a^2 = b^2 + c^2 - 2c(bCosA)$$

$$\therefore \boldsymbol{a^2 = b^2 + c^2 - 2bcCosA}$$

Therefore for finding sides we use: $a^2 = b^2 + c^2 - 2bcCosA$

For angles, we make $CosA$ the subject.

$a^2 = b^2 + c^2 - 2bcCosA$

$\Longrightarrow 2bcCosA = b^2 + c^2 - a^2$

$\therefore \boldsymbol{CosA = \frac{b^2 + c^2 - a^2}{2bc}}$

NB: $Cos\,\theta \ \ \{\boldsymbol{acute}\} = -Cos\,(180 - \theta) \ \ \{\boldsymbol{obtuse}\}$

WORKED EXAMPLES

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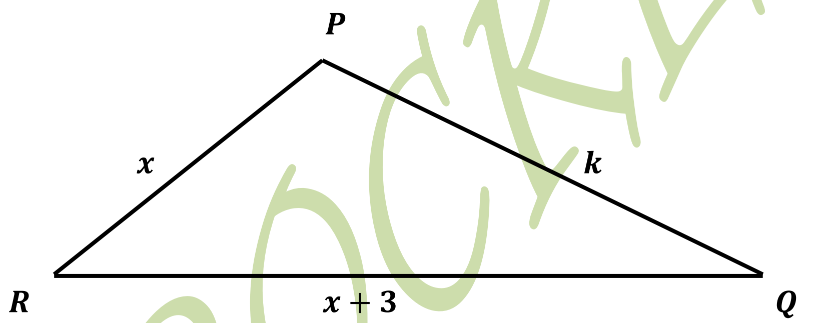

The diagram shows triangle PQR in which $PQ = k\ units$, $QR = (x + 3)\ units$ and $RP = x\ units$.

Express $cosR$ in terms of x and k. **[1]**

Given that $cosR = \frac{1}{2}$, show that $x^2 + 3x + 9 - k^2 = 0$. **[2]**

Hence find the value of k for which there is only one possible triangle. **[2]**

Solution

$CosR = \frac{p^2+q^2-r^2}{2pq}$

$CosR = \frac{x^2+(x+3)^2-k^2}{2(x)(x+3)}$

$CosR = \frac{x^2+x^2+6x+9-k^2}{2x^2+6x}$

$CosR = \frac{2x^2+6x+9-k^2}{2x^2+6x}$

Given:

$CosR = \frac{1}{2}$

$\Longrightarrow \frac{2x^2+6x+9-k^2}{2x^2+6x} = \frac{1}{2}$

$\Longrightarrow 2(2x^2+6x+9-k^2) = 1(2x^2+6x)$

$\Longrightarrow 4x^2+12x+18-2k^2) = 2x^2+6x$

$\Longrightarrow 2x^2+6x+18-2k^2 = 0$

$\therefore x^2+3x+9-k^2 = 0$ (as required)

Finding the value of k for which there is only one possible triangle:

The value of x should be the same in order to have only one possible triangle.

Thus $b^2-4ac = 0$

$\Longrightarrow 3^2-4(1)(9-k^2) = 0$

$\Longrightarrow 9-36+4k^2 = 0$

$\Longrightarrow -27+4k^2 = 0$

$\Longrightarrow 4k^2 = 27$

$\Longrightarrow k^2 = \frac{27}{4}$

$\Longrightarrow k = \sqrt{\frac{27}{4}}$

$\Longrightarrow k = \sqrt{\frac{9\times 3}{4}}$

$\therefore k = \frac{3}{2}\sqrt{3}$

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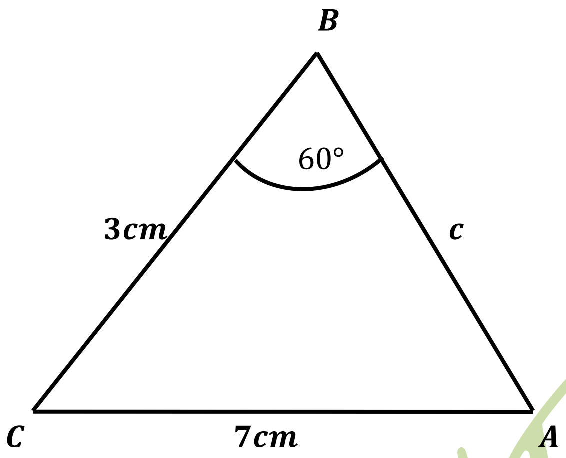

Given that in triangle , $AC = 7cm$, $CB = 7cm$ and $ABC = 60°$, show that $CosC = -\frac{1}{7}$. **[5]**

Solution

$$\frac{SinA}{a} = \frac{Sin\,B}{b}$$

$$\frac{SinA}{3} = \frac{Sin\,(60)}{7}$$

$$Sin\ A = \frac{3Sin\left(\frac{\pi}{3}\right)}{7}$$

$$Sin\ A = \frac{3\left(\frac{\sqrt{3}}{2}\right)}{7}$$

$$A = Sin^{-1}\left(\frac{3\sqrt{3}}{14}\right)$$

$$A = 21.7867893$$

Now $C = 180° - (60° + 21.78617893°) = 98.2132107°$ and taking $Cosines$ on both sides:

$$CosC = Cos(98.2132107°)$$

$$CosC = -0.142857142 = -\frac{1}{7}$$

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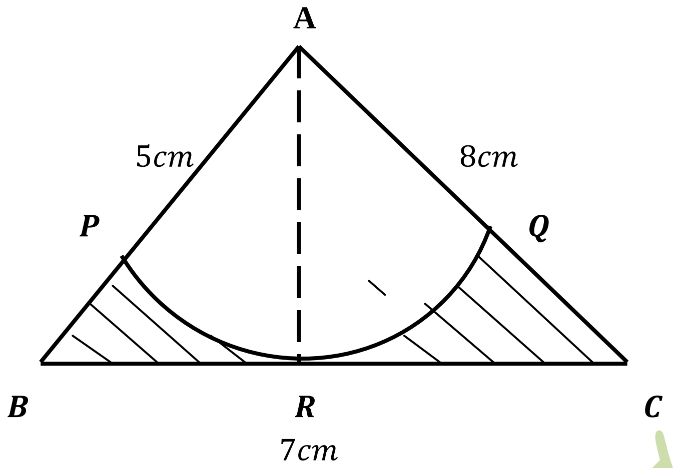


Triangle ABC is such that $AB = 5cm$, $BC = 7cm$ and $CA = 8cm$. A circular arc PQ is drawn with centre A and touches the line BC at R.

Show that

1. angle $BAC = \frac{\pi}{3}$,
2. $Sin\ B = \frac{4}{7}\sqrt{3}$. **[4]**

Solution

1. $CosA = \frac{b^2+c^2-a^2}{2bc}$

$CosA = \frac{8^2+5^2-7^2}{2(8)(5)}$

$CosA = \frac{64+25-49}{80}$

$CosA = \frac{40}{80}$

$CosA = \frac{1}{2}$

$A = Cos^{-1}\left(\frac{1}{2}\right)$

$A = \frac{\pi}{3}$

$\therefore$angle $= \frac{\pi}{3}$.

2. $\frac{SinB}{b} = \frac{Sin\ A}{a}$

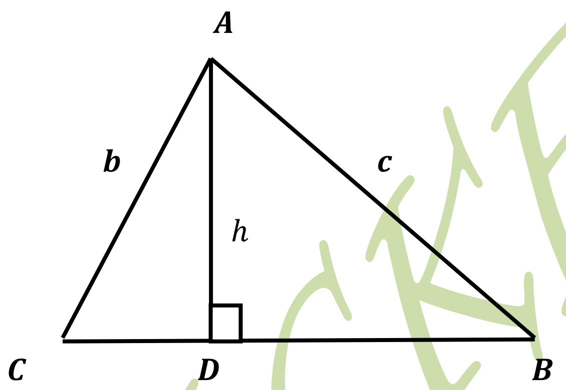
$\frac{SinB}{8} = \frac{Sin\left(\frac{\pi}{3}\right)}{7}$

$$Sin\ B = \frac{8Sin\left(\frac{\pi}{3}\right)}{7}$$

$$Sin\ B = \frac{8\left(\frac{\sqrt{3}}{2}\right)}{7}$$

$$Sin\ B = \frac{4\sqrt{3}}{7}$$

AREA OF TRIANGLE



Area of triangle $ACB = \frac{1}{2} \times base \times perpendicular\ height$

$= \frac{1}{2} \times a \times h$ (i)

Now using triangle ACD:

$$sin\theta = \frac{Opposite}{Hypotenuse}$$

$$sinC = \frac{h}{b}$$

$\Rightarrow h = bsinC$ (ii)

Substituting equation (ii) into equation (i):

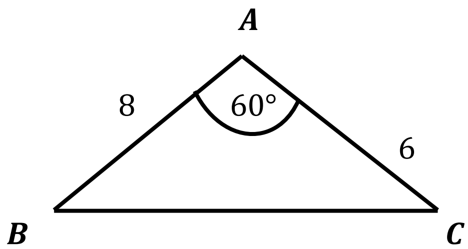
Area of triangle $ACB = \frac{1}{2} \times a \times h$

$\Rightarrow$Area of triangle $ACB = \frac{1}{2} \times a \times bsinC$

$\therefore$Area of triangle $ACB = \frac{1}{2}absinC$

WORKED EXAMPLE

Find the exact value of the area of triangle ABC.



Solution

Area of triangle $ACB = \frac{1}{2}absinC$

$$= \frac{1}{2}(8)(6)sin60$$

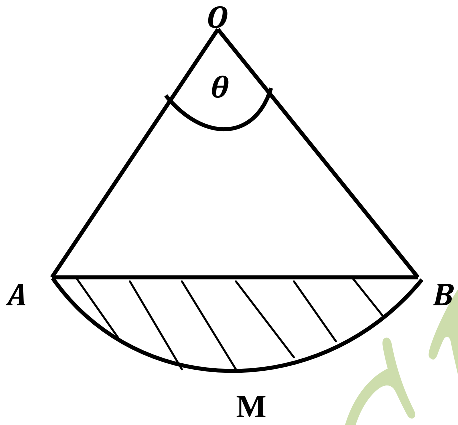
$$= 24\left(\frac{\sqrt{3}}{2}\right)$$

$\therefore$ Area of triangle $ACB = 12\sqrt{3}$

SECTORS & SEGMENTS

SECTOR

A sector is a part of a circle extending to the centre.



Area of sector $= \frac{\theta}{360} \times area\ of\ circle$

$= \frac{\theta}{360} \times \pi r^2$

But $360° = 2\pi$

$= \frac{\theta}{2\pi} \times \pi r^2$

$= \frac{\theta}{2} \times r^2$

$\therefore$ **Area of sector** $= \frac{1}{2} r^2 \theta$

LENGTH OF ARC

Length of arc $= \frac{\theta}{360} \times circumference$

$= \frac{\theta}{360} \times 2\pi r$

But $360° = 2\pi$

$$= \frac{\theta}{2\pi} \times 2\pi r$$

$\therefore$ Length of arc $= r\theta$

SEGMENT

Area of triangle $OAB = \frac{1}{2} abSinO = \frac{1}{2} r^2 Sin\theta$

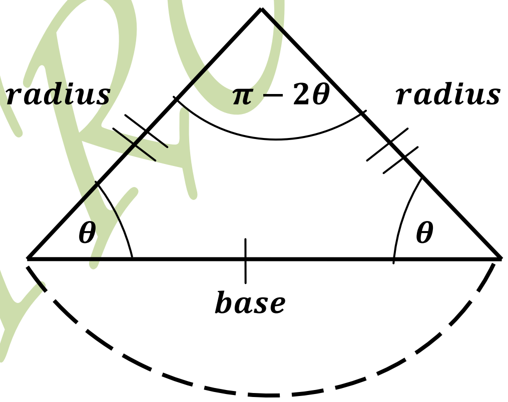
Now area of segment (shaded part) $=$ $Area\ of\ sector - Area\ of\ triangle$

$$= \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 Sin\theta$$

$\therefore$**Now area of segment (shaded part) $= \frac{1}{2} r^2 (\theta - Sin\theta)$**

IMPORTANT POINTS

The triangle on a Sector



- It is an isosceles triangle
- The base angles are equal
- The angles in a triangle add up to π or $180°$

Double Angles and Compound Angles

Remember the following identies when evaluating some problems on sectors:

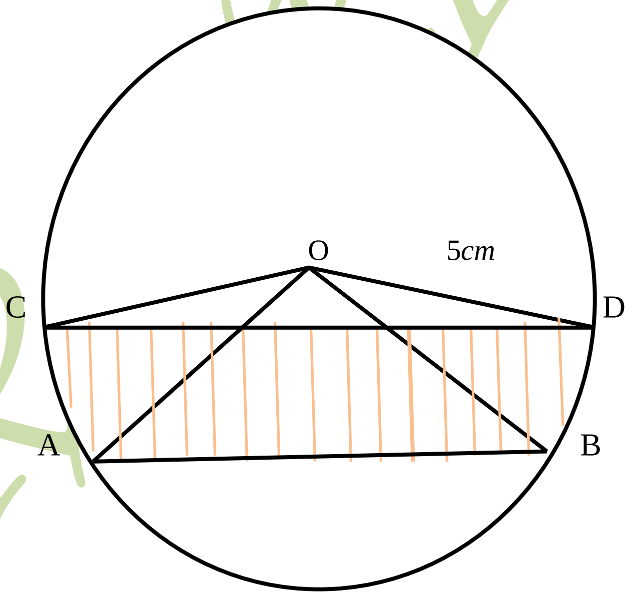
- $Sin\ (A \pm B) = SinACosB \pm CosASinB$
- $Sin2A = Sin\ (A + A) = SinACosA + CosASinA = 2SinACosA$
- $Cos\ (A \pm B) = CosACosB \mp SinASinB$
- $Cos2A = Cos(A + A) = CosACosA - SinASinA$

$$= Cos^2A - Sin^2A$$
$$= 2Cos^2A - 1 \quad \text{or} \quad 1 - 2Sin^2A$$

Remember that $Cos^2A + Sin^2A = 1$.

WORKED EXAMPLES

ZIMSEC NOVEMBER 2003 PAPER 1



The diagram above shows a circle centre O and radius $5cm$. The chord AB is parallel to the chord CD. The angles $A\hat{O}B$ and $C\hat{O}D$ are θ and $\left(\theta + \frac{\pi}{2}\right) radians$ respectively. Show that the area of the shaded region is

$$\frac{25}{4}\left[\pi + 2\sqrt{2}\sin\left(\theta - \frac{\pi}{4}\right)\right]$$

[6]

Solution

Shaded Area = Area of Major Segment CD – Area of Minor Segment AB

Aside

$$Area\ of\ Segment = \frac{1}{2}r^2(\theta - sin\theta)$$

Therefore:

$$Area\ of\ Major\ Segment\ CD = \frac{1}{2}(5)^2\left[\left(\theta + \frac{\pi}{2}\right) - sin\left(\theta + \frac{\pi}{2}\right)\right]$$

$$= \frac{25}{2}\left[\theta + \frac{\pi}{2} - \left(sin\theta cos\frac{\pi}{2} + cos\theta sin\frac{\pi}{2}\right)\right]$$

$$= \frac{25}{2}\left(\theta + \frac{\pi}{2} - cos\theta\right)$$

Also:

$$Area\ of\ Major\ Segment\ CD = \frac{1}{2}(5)^2[\theta - sin\theta]$$

$$= \frac{25}{2}(\theta - sin\theta)$$

Now:

$$Shaded\ Area = \frac{25}{2}\left(\theta + \frac{\pi}{2} - cos\theta\right) - \frac{25}{2}(\theta - sin\theta)$$

$$= \frac{25}{2}\theta + \frac{25\pi}{4} - \frac{25}{2}cos\theta - \frac{25}{2}\theta + \frac{25}{2}sin\theta$$

$$= \frac{25\pi}{4} + \frac{25}{2}sin\theta - \frac{25}{2}cos\theta$$

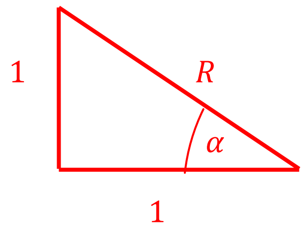
$$= \frac{25\pi}{4} + \frac{25}{2}(sin\theta - cos\theta) \qquad \text{(i)}$$

Now expressing $sin\theta - cos\theta$ in the form $Rsin(\theta - \alpha)$:

$$sin\theta - cos\theta \equiv Rsin(\theta - \alpha)$$

$$sin\theta - cos\theta \equiv Rsin\theta cos\alpha - Rcos\theta sin\alpha$$

$\Longrightarrow Rcos\alpha = 1 \Longrightarrow cos\alpha = \frac{1}{R}$ and $Rsin\alpha = 1 \Longrightarrow sin\alpha = \frac{1}{R}$



Now using the Pythagoras' theorem: $R^2 = 1^2 + 1^2 = 2 \therefore R = \sqrt{2}$

From Trigonometric ratios: $Tan\alpha = \frac{1}{1} = 1 \therefore \alpha = Tan^{-1}(1) = \frac{\pi}{4}$

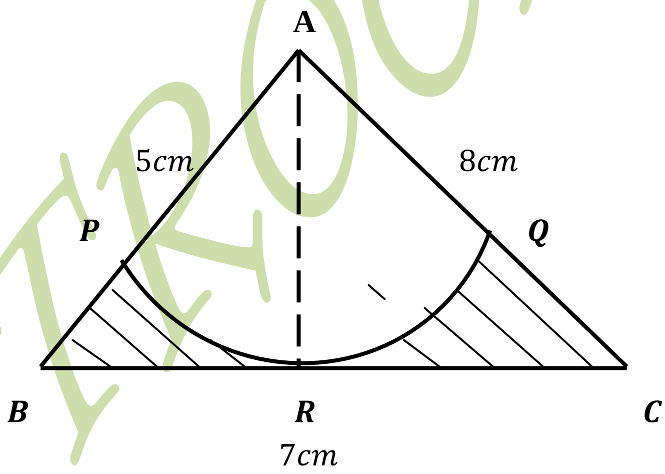
$\therefore sin\theta - cos\theta \equiv \sqrt{2}sin\left(\theta - \frac{\pi}{4}\right)$ (ii)

Now substituting (ii) into (i):

$$Shaded\ Area = \frac{25\pi}{4} + \frac{25}{2}\left[\sqrt{2}sin\left(\theta - \frac{\pi}{4}\right)\right]$$

$$= \frac{25}{4}\left[\pi + 2\sqrt{2}sin\left(\theta - \frac{\pi}{4}\right)\right] \quad (as\ required)$$

<u>Ministry of Education and Cambridge NOVEMBER 1997 and ZIMSEC JUNE 2014</u>



Triangle ABC is such that $AB = 5cm$, $BC = 7cm$ and $CA = 8cm$. A circular arc PQ is drawn with centre A and touches the line BC at R.

a. Show that

1. angle $BAC = \frac{\pi}{3}$,

2. $Sin\ B = \frac{4}{7}\sqrt{3}$. **[4]**

b. Find the exact area of the shaded region. (**June 2014**) **[3]**

NB: The same question was examined in November 1997 and it was phrased as:

Show that the area of the shaded region, which lies inside the triangle ABC but outside sector APQ, is

$$\left(10\sqrt{3} - \frac{200}{49}\pi\right) cm^2.$$ [**5**]

Solution

(a)

1. $CosA = \frac{b^2+c^2-a^2}{2bc}$

$CosA = \frac{8^2+5^2-7^2}{2(8)(5)}$

$CosA = \frac{64+25-49}{80}$

$CosA = \frac{40}{80}$

$CosA = \frac{1}{2}$

$A = Cos^{-1}\ \left(\frac{1}{2}\right)$

$A = \frac{\pi}{3}$

$\therefore$angle $= \frac{\pi}{3}$ $\quad (as\ required).$

2. $\frac{SinB}{b} = \frac{Sin\ A}{a}$

$\frac{SinB}{8} = \frac{Sin\left(\frac{\pi}{3}\right)}{7}$

$Sin\ B = \frac{8Sin\left(\frac{\pi}{3}\right)}{7}$

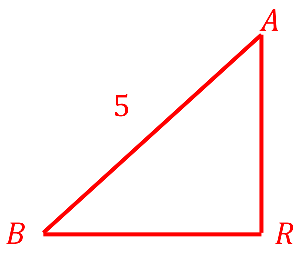
$Sin\ B = \frac{8\left(\frac{\sqrt{3}}{2}\right)}{7}$

$Sin\ B = \frac{4\sqrt{3}}{7} \quad (as\ required)$

b) $Shaded\ Area = Area\ triangle\ ABC - Area\ of\ sector\ APRQ$

Aside

Finding the length of the radius of the sector:



$Sin\alpha = \frac{AR}{5}$

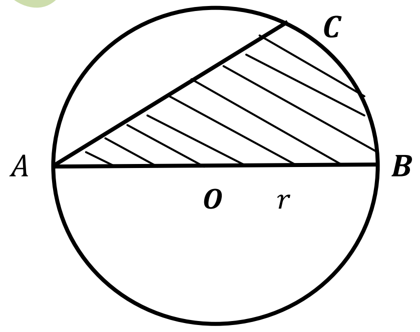
$\therefore AR = 5SinB = 5\left(\frac{4}{7}\sqrt{3}\right) = \frac{20}{7}\sqrt{3}$

$Area\ of\ sector\ APRQ = \frac{1}{2}r^2\theta = \frac{1}{2}\left(\frac{20}{7}\sqrt{3}\right)^2\left(\frac{\pi}{3}\right) = \frac{1}{2}\left(\frac{400}{49}\right)(3)\left(\frac{\pi}{3}\right) = \frac{200\pi}{49}$

Also $Area\ triangle\ ABC = \frac{1}{2}abSin\theta = \frac{1}{2}(5)(7)\left[\frac{4}{7}\sqrt{3}\right] = 10\sqrt{3}$

$\therefore Shaded\ Area = \left(10\sqrt{3} - \frac{200\pi}{49}\right)cm^2 \qquad (as\ required)$

ZIMSEC NOVEMBER 2011 PAPER 1



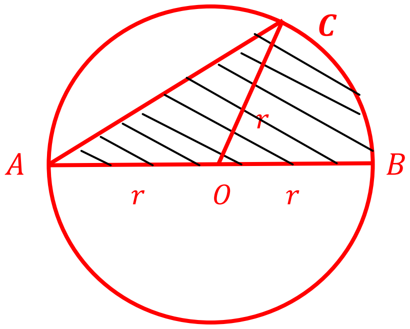
The above diagram shows a circle of radius rcm and centre O. The shaded region is bounded by the chord AC, the diameter AB and the arc BC, and the angle $B\hat{A}C = \theta$ radians.

a) Find an exact expression in terms of r and θ for the perimeter of the shaded region.

[3]

b) Given that $\theta = \frac{\pi}{6}$, calculate the exact area of the shaded region in terms of r. **[4]**

Solution



a) Make the annotations on the diagram as illustrated above and take note of the following:

$B\hat{A}C = \theta \equiv O\hat{C}A = \theta$; $A\hat{O}C = \pi - 2\theta$ and $C\hat{O}B = 2\theta$. Also $OA = OB = OC = r$.

Now:

$Perimeter = AC + arcBC + AB$

Aside

$AC^2 = r^2 + r^2 - 2r^2cos(\pi - 2\theta\ )$

$= 2r^2 - 2r^2(cos\pi cos2\theta - sin2\pi sin2\theta)\ )$

$= 2r^2 - 2r^2(-cos2\theta)$

$= 2r^2 + 2r^2cos2\theta$

$= 2r^2 + 2r^2(2cos^2\theta - 1)$

$= 2r^2 + 4r^2cos^2\theta - 2r^2$

$AC^2\ = 4r^2cos^2\theta$

$AC\ = \sqrt{4r^2cos^2\theta}$

$\therefore AC\ = 2rcos\theta$

$\Longrightarrow Perimeter = 2rcos\theta + 2r\theta + 2r = 2r(cos\theta + \theta + 1)$

b) $Area\ of\ the\ shaded\ region = Area\ of\ semicircle - Area\ of\ segment\ AC$

Aside

$$Area\ of\ segment\ AC = \frac{1}{2}r^2[(\pi - 2\theta) - sin(\pi - 2\theta)]$$

$$= \frac{1}{2}r^2[\pi - 2\theta - (sin\pi cos2\theta - cos\pi sin2\theta)]$$

$$= \frac{1}{2}r^2(\pi - 2\theta + sin2\theta)$$

Now:

$$Area\ of\ the\ shaded\ region = \frac{1}{2}\pi r^2 - \frac{1}{2}r^2(\pi - 2\theta + sin2\theta)$$

$$= \frac{1}{2}\pi r^2 - \frac{1}{2}\pi r^2 + \frac{1}{2}r^2(2\theta) - \frac{1}{2}r^2 sin2\theta$$

$$= \frac{1}{2}r^2(2\theta - sin2\theta)$$

But $\theta = \frac{\pi}{6}$

$$\therefore Area\ of\ shaded\ region\ = \frac{1}{2}r^2\left[2\left(\frac{\pi}{6}\right) - sin2\left(\frac{\pi}{6}\right)\right]$$

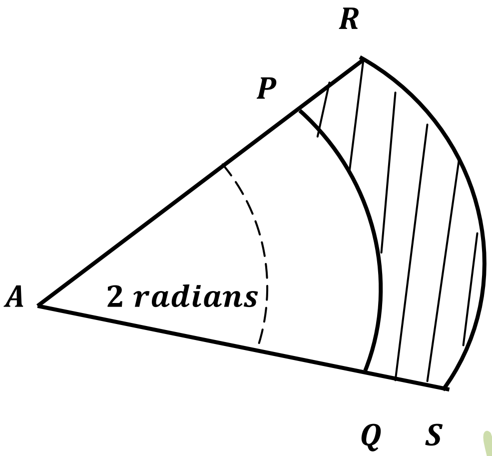
$$= \frac{1}{2}r^2\left[\left(\frac{\pi}{3}\right) - sin\left(\frac{\pi}{3}\right)\right]$$

$$= \frac{1}{2}r^2\left[\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right]$$

$$= \frac{1}{2}r^2\left[\frac{2\pi - 3\sqrt{3}}{6}\right]$$

$$= \frac{1}{12}r^2(2\pi - 3\sqrt{3})$$

ZIMSEC NOVEMBER 2013 PAPER 1



In the diagram above, the shaded region is bounded by sectors APQ and ARS. It is given that the angle PAQ is 2 $radians$ and the perimeter of $PQSR$ is 32cm.

(i) Calculate the length of AS.

(ii) If the area of $PQSR$ is 28 cm^2, find the of AQ. **[5]**

Solutions

(i) Perimeter of PQSR is 32cm.

$Perimeter = PR + QS + arc\ PQ + arcRS$

Aside

$QS = PR$ and also $AQ + QS = AS$

$\therefore 2QS + 2AQ + 2AS = 32$

$2(QS + AQ) + 2AS = 32$

$2AS + 2AS = 32$

$4AS = 32$

$\therefore AS = 8cm.$

(ii) Area of $PQSR$ is 28 cm^2.

$Shaded\ Area = Area\ of\ sectorARS - Area\ of\ sector\ APQ$

$\frac{1}{2}(AS)^2(2) - \frac{1}{2}(AQ)^2(2) = 28$

$(8)^2 - (AQ)^2 = 28$

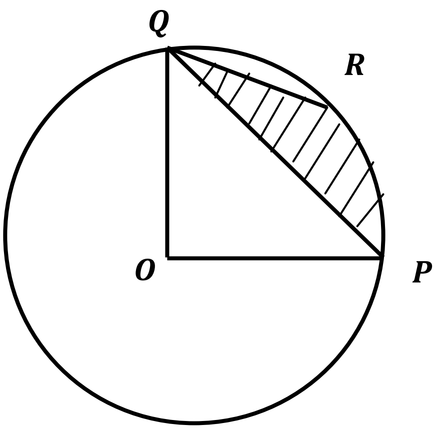
$(AQ)^2 = 64 - 28$

$(AQ)^2 = 36$

$AQ = \sqrt{36}$

$\therefore AQ = 6cm$

ZIMSEC NOVEMBER 2015 PAPER 1



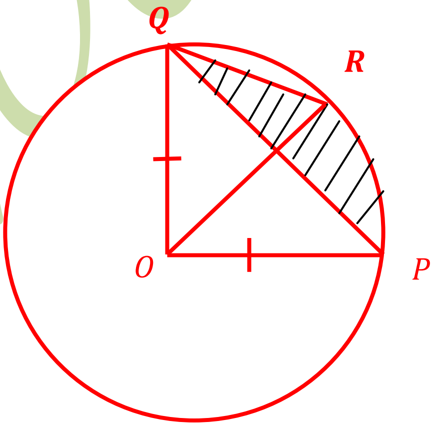
The diagram shows a circle O and radius r.

A chord PQ subtends are right-angle at the centre O of the circle, QR is a chord such that angle PQR is $\frac{\pi}{12}$.

Show that the shaded area bounded by chords PQ and QR, and the arc PR is

$$\frac{1}{2}r^2\left(\frac{\pi}{6}+\frac{1}{2}\sqrt{3}-1\right).$$ [**5**]

Solutions



Make the annotations on the diagram as illustrated above and take note of the following:

(i) Triangle AQR is isosceles thus $OQ = OP = r$.
(ii) $Q\hat{O}P = \frac{\pi}{2} \Longrightarrow O\hat{Q}P = O\hat{P}Q = \frac{\pi}{4}$.

(iii)Angle $PQR = \frac{\pi}{12} \Longrightarrow O\hat{Q}R = \frac{\pi}{12} + \frac{\pi}{4} = \frac{\pi}{3} \Longrightarrow O\hat{Q}R = O\hat{R}Q = Q\hat{O}R = \frac{\pi}{3}$.

(iv)Also $OQ = OR = OP = r$.

Now:

$Shaded\ area = \ Area\ of\ segmentQRP - \ Area\ of\ segmentQR$

$$= \frac{1}{2}r^2\left[\frac{\pi}{2} - Sin\left(\frac{\pi}{2}\right)\right] - \frac{1}{2}r^2\left[\frac{\pi}{3} - Sin\left(\frac{\pi}{3}\right)\right]$$

$$= \frac{1}{2}r^2\left[\frac{\pi}{2} - 1 - \frac{\pi}{3} + Sin\left(\frac{\pi}{3}\right)\right]$$

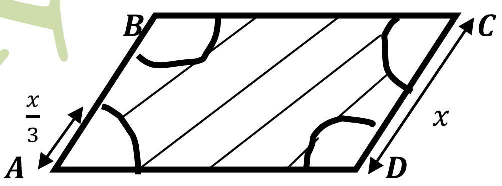
$$= \frac{1}{2}r^2\left(\frac{\pi}{2} - \frac{\pi}{3} - 1 + \frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{2}r^2\left(\frac{\pi}{6} + \frac{1}{2}\sqrt{3} - 1\right) \qquad (as\ required)$$

Follow Up Exercise

Question 1

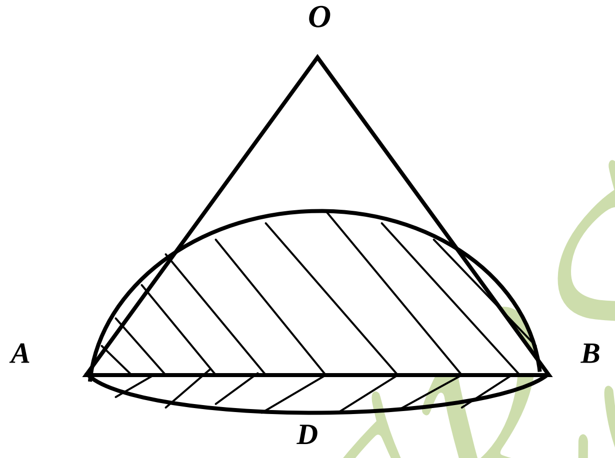
The diagram below, $ABCD$, is a rhombus of side x and $\hat{A} = \theta$ radians



Arcs each of radius $\frac{x}{3}$ are drawn with centres A, B, C and D. If the shaded area is half the area of the rhombus, show that $sin\theta = \frac{2\pi}{9}$ and find the 2 possible values of θ.

Question 2

The diagram shows an equilateral triangle OAB of side $10\,cm$. The arc ADB is drawn with centre O.



A semi-circle is drawn on AB as diameter.

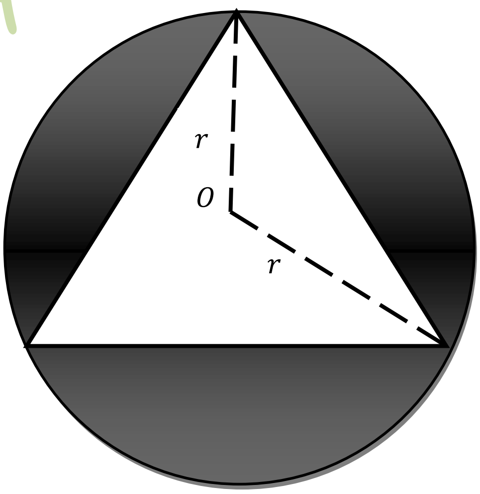
i) State the angle $A\hat{O}B$ in $radians$,

ii) Find the area of the shaded region, give your answer correct to 2 $d.p$.

iii) Find the fraction of the area of shaded segment ADB to the area of triangle OAB, give your answer correct to 1 $s.f$.

Question 3

The diagram below shows an equilateral triangle ABC whose vertices lie on a circle centre O, of radius r.

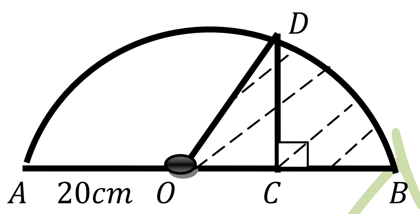


Show that

a) the length of a side of triangle is $r\sqrt{3}$.

b) the ratio of the area of the shaded region to the area of the triangle is $\left(4\pi\sqrt{3}-9\right):9$

Question 4

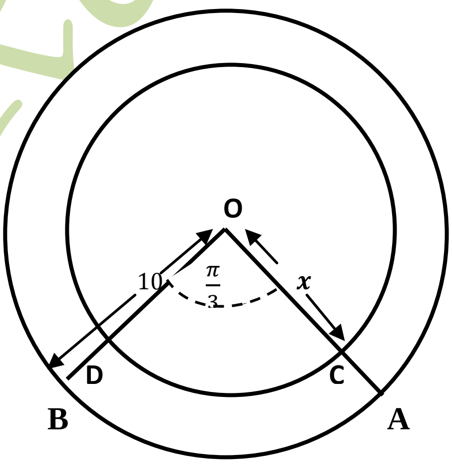


In the diagram above, ADB is a semicircle with centre O and radius $20cm$. DC is perpendicular to AB where C is the midpoint of OB. Calculate

(i) angle $D\hat{O}C$ in radians,

(ii) the area of the shaded region.

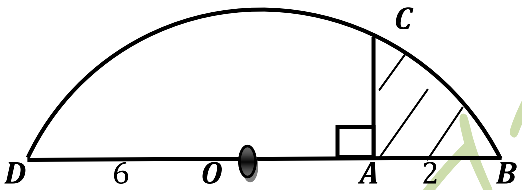
Question 5



The diagram shows two circles with centre O. $OA = OB = 10cm$ and $OC = OD = xcm$. The angle AOB is $\frac{\pi}{3}$. The difference between the area of the minor sectors OAB and OCD is 6π. Find the value of x.

Question 6

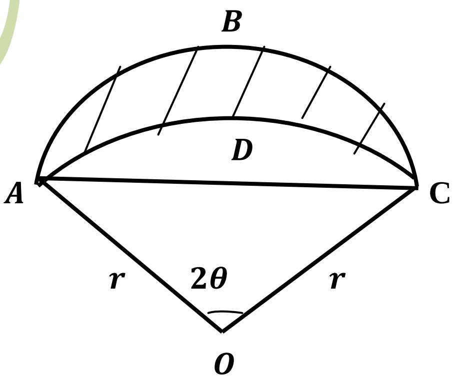
In the diagram below, $DOBC$ is a semicircle, center O and radius $6cm$. AC is perpendicular to DOB where $AB = 2cm$.



Calculate

a) The exact length of AC.
b) Angle $C\hat{O}A$ in radians.
c) The perimeter of the shaded region.
d) Express the area of the shaded region as a percentage of the area of the semicircle.

Question 7



The diagram above shows a sector $OABC$ with arc ADC, centre O, radius r and $A\hat{O}C = 2\theta$ $radians$. ABC is a semi circle on AC as diameter. Show that $AC = 2r\sin\theta$.

Find expressions, in terms of r and θ, for the areas of

(i) the sector $OADC$,

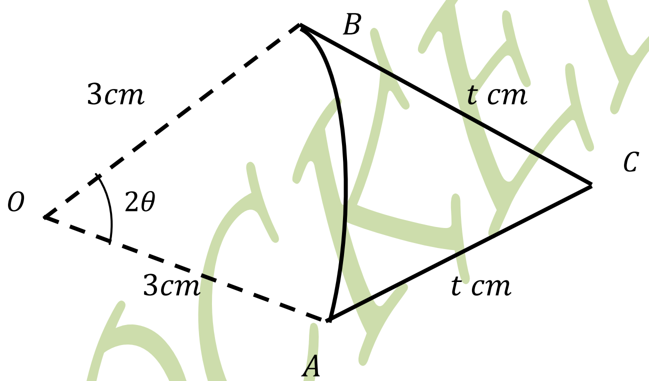
(ii) the segment ADC,

(iii)the shaded region.

Question 8

A sector of a circle radius r has a total perimeter of $12cm$. If its area is Acm^2, show that $A = 6r - r^2$.Hence find the value of r for which A is a maximum and the corresponding value of the angle of sector in radians.

Question 9

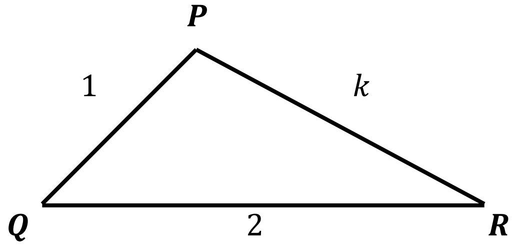


The diagram shows the shape ABC formed from a piece of wire $10cm$ in length. AB is the arc of a circle centre O and radius $3cm$. CA and CB are tangents to the circle , each of length $t\ cm$.

Given that the angle subtended by the arc at the centre is 2 θ radians, show that $t = 5 - 3\theta$

Hence show that $t = 5 - 3\tan^{-1}\left(\frac{t}{3}\right)$.

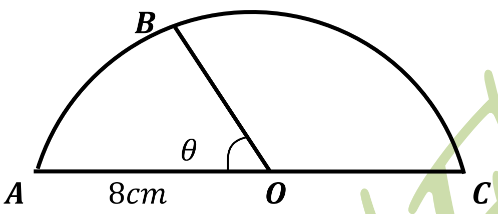
Question 10



The diagram above shows triangle PQR, in which $PQ = 1$ unit, $QR = 2$ units and $RP = k$ units. Express $cosR$ in terms of k.

Given that $cosR < \frac{7}{8}$, show that $2k^2 - 7k + 6 < 0$. Find the set of values of k satisfying this inequality.

Question 11



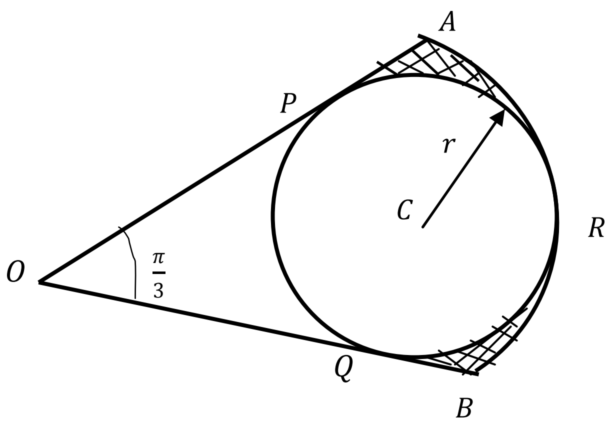
The diagram shows a semi-circle ABC with centre O and radius $8cm$. Angle $= \theta\ radians$..

(i) In the case where $\theta = 1$, calculate the area of the sector BOC.

(ii) Find the value of θ for which the length of arc AB is one half of the length of arc BC.

(iii)By using the value of θ in 6(ii) above, show that the exact length of the perimeter of triangle ABC is $(24 + 8\sqrt{3})cm$.

Question 12



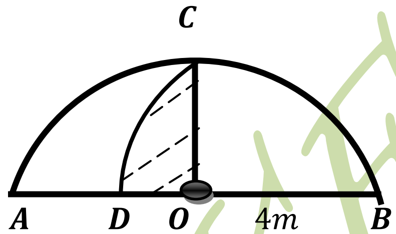
The diagram above shows a sector OAB of a circle with centre O and radius 24 cm. The circle PQR with centre C and radius r cm is inscribed in the sector.
Show that $r = 8cm$ and $OQ = 8\sqrt{3}$ cm.
Calculate, in exact form,

(a) the area of sector OAB,

(b) the area of the quadrilateral $OQCP$.

Hence show that the area of the shaded region is $\frac{32}{3}(5\pi - 6\sqrt{3})$ cm^2.

Question 13

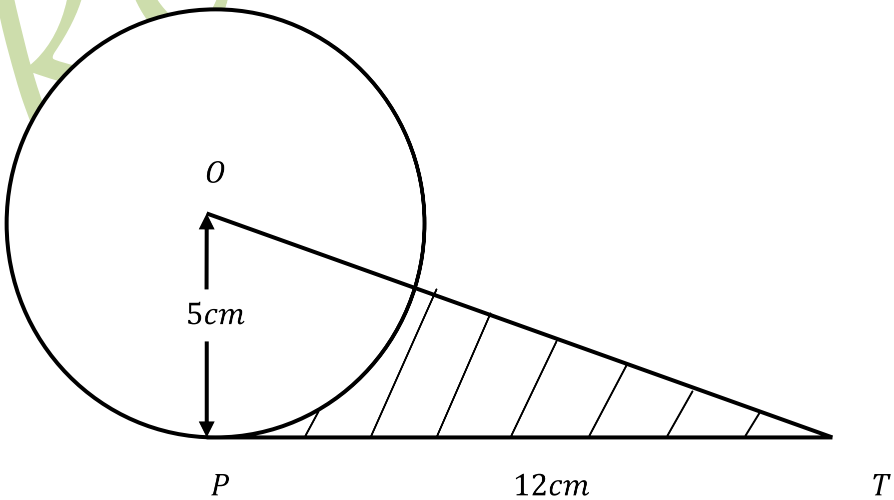


The diagram shows a semicircle ABC with center O and radius $4m$, such that the angle $B\hat{O}C = 90°$. Given that CD is an arc of a circle, centre B, calculate

(i) the length of arc CD,

(ii) the area of the shaded region.

Question 14



The diagram shows a circle with centre O and radius 5 cm. The point P lies on the circle, PT is a tangent to the circle and $PT = 12$ cm. The line OT cuts the circle at the point Q.

(a) Find the perimeter of the shaded region.

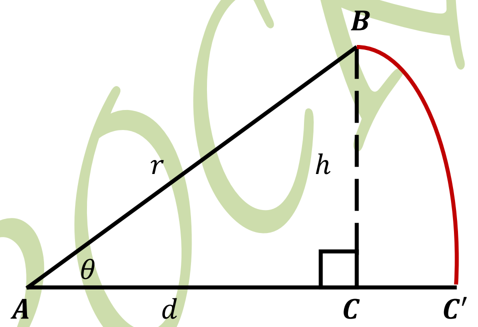
(b) Find the area of the shaded region.

Small angle approximations for $Sin\ x$, $Cos\ x$ and $Tan\ x$

Angles less than $15°$ are considered as small angles

Geometrical Derivation of the Small Angle Approximations

Geometrically, we can derive the small angle approximation using trigonometric ratios.



On the right angled triangle ABC above the perpendicular height h can be calculated using trigonometric ratios as follows:

$tan\ (\theta) = \frac{h}{d} \quad \Rightarrow h = d\ tan\ (\theta)$ and $sin\ (\theta) = \frac{h}{r} \quad \Rightarrow h = r\ sin\ (\theta)$

As the angle θ becomes close to zero then $r \approx d$.

In addition, the height h becomes closer to the length of the circular arc joining $B\ to\ C'$, which can be calculated as $r\theta$ (provided the angle is given in radians).

Hence, we have

$$h = r\,sin\,(\theta) \approx r\theta \approx r\,tan\,(\theta)$$

This leads to the approximations

$$sin\,(\theta) \approx tan\,(\theta) \approx \theta$$

We can use the double angle formula obtain an approximation for $y = cos(\theta)$ as follows:

$$cos(2x) = 1 - 2sin^2(x);$$

with $x = \frac{\theta}{2}$ and apply the small angle approximation for $sin(x)$.

Hence,

$$cos(\theta) = 1 - 2sin^2\frac{\theta}{2}$$

$$\approx 1 - 2\left(\frac{\theta}{2}\right)^2$$

$$= 1 - \frac{\theta^2}{2}$$

Alternatively, the trigonometric functions can be expressed using their Taylor Series approximations. These are infinite power series which get increasingly close to the value of the underlying function as more terms are included.

Taylor Series expansions about the point $x = 0$ for the three common trigonometric ratios $Sin\,x$, $Cos\,x$ and $Tan\,x$ are:

$$sin\,(\theta) \approx \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots$$

$$tan\,(\theta) \approx \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots$$

$$cos\,(\theta) \approx 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots$$

Since the value of θ is small, thus θ^3 and higher powers of θ becomes very insignificant (they approach zero).

Hence the approximations become:

$$sin\,(\theta) \approx \theta$$

$$tan\ (\theta) \approx \theta$$

$$cos\ (\theta) \approx 1 - \frac{\theta^2}{2!}$$

Small Angle Approximations

For θ close to zero and measured in radians, the small angle approximations are:

$$\boldsymbol{sin\ (\theta) \approx \theta}$$

$$\boldsymbol{tan\ (\theta) \approx \theta}$$

$$\boldsymbol{cos\ (\theta) \approx 1 - \frac{\theta^2}{2}}$$

WORKED EXAMPLES

Example

Show that, for small angles, the function $f\ (x) = sin^2\ (x)\ \ cos\ (x)$ can be approximated by a function of the form $h(x) = a + bx + cx^2 + dx^3 + ex^4$ and use this approximation to evaluate $sin^2\left(\frac{\pi}{24}\right)\ \ cos\left(\frac{\pi}{24}\right)$

Solution

$$sin^2\ (x)\ cos\ (x) \approx (x)^2\left(1 - \frac{x^2}{2}\right)$$

$$= x^2 - \frac{x^4}{2}$$

The above function is of the desired form with $a = b = d = 0, c = 1$ and $e = \frac{1}{2}$

Using this approximation:

$$\sin^2\left(\frac{\pi}{24}\right)\ \cos\left(\frac{\pi}{24}\right)\ \approx\ \left(\frac{\pi}{24}\right)^2-\frac{\left(\frac{\pi}{24}\right)^4}{2}$$

$$\approx 0.01699$$

Example

(a) When x is small, show that $tan\ (3x)\ cos\ (2x)$ can be approximated by $3x - 6x^3$

(b) Hence, approximate the value of $tan\ (0.2)\ cos\ (0.2)$

(c) Calculate the percentage error in your approximation

Solution

a) $tan\ (3x) cos\ (2x) = 3x\left[1-\frac{(2x)^2}{2}\right]$

$$= 3x\left(1-\frac{4x^2}{2}\right)$$

$$= 3x(1-2x^2)$$

$$= 3x - 6x^3 \qquad (as\ required)$$

b) $3x = 0.2 \Longrightarrow x = 0.066666666$

$\therefore tan\ (0.2) cos\ (0.2) = 3(0.066666666) - 6(0.066666666)^3$

$$= 0.198222222$$

c) $Percentage\ error = \frac{|Actual - Estimate|}{Actual} \times 100\%.$

Aside

$Actual\ value = tan\ (0.2) cos\ (0.2) = 0.19866933$ (Use radians)

Therefore:

$$Percentage\ error = \frac{|0.19866933\ - 0.198222222|}{0.19866933} \times 100\%$$

$$= 0.002250523 \times 100\%$$

$$= 0.2250523\%$$

$$= 0.23\% \quad (to\ 2s.f.)$$

Example

Find a polynomial approximation of the function $f\,(\theta) = \; cos^{\,2}(\theta)sin^2\,(\theta)$

Solution

$f\,(\theta) = \; cos^{\,2}(\theta)sin^2\,(\theta) \; = [cos(\theta)]^{\,2}[sin(\theta)]^2$

$$= \left[1 - \frac{(\theta)^2}{2}\right]^2 (\theta)^2$$

$$= \left[1 - (\theta)^2 + \frac{(\theta)^4}{4}\right] (\theta)^2$$

$$= (\theta)^2 - (\theta)^4 + \frac{(\theta)^6}{4}$$

FOLLOW UP EXERCISE

Question 1

(a) Given that θ is small, use the small angle approximation of $cos\ \theta$ to show that

$$4\ cos\ (\theta)\ +\ cos^2\ (2\theta) \approx\ 5 - 6\theta^2\ +\ 4\theta^4$$

(b) Hence find an approximation of $4\ cos\ (\theta)\ +\ cos^2\ (2\theta)$ when $\theta\ =\ 3°$

(c) Calculate the percentage error in your approximation

Question 2

Show that when θ is small the approximate value of $\frac{1-\cos 2\theta}{\theta tan\theta}$ is 2.

Question 3

Approximate $sin\ (\theta), cos\ (\theta), tan\ (\theta)$ using the small angle approximations for

(i) $\theta =\ 15°$

(ii) $\theta = 3°$

(iii) $\theta = \frac{\pi}{32}$

(iv) $\theta = \frac{\pi}{100}$

Question 4

a) When θ is show that $\frac{Cos\theta}{Sin\theta}$ can be approximated as $\frac{2-\theta^2}{2\theta}$

b) Hence approximate the value of $\frac{Cos\left(\frac{\pi}{24}\right)}{Sin\left(\frac{\pi}{24}\right)}$

c) Calculate the percentage error in your approximation.

Question 5

Find a polynomial approximation of the function:

(i) $f(\theta) = cos^2(\theta)\tan(\theta)$,

(ii) $f(\theta) = sec(\theta)sin(\theta) - cos(\theta)$

(iii) $f(\theta) = \frac{1-cos(\theta)}{\theta^2}$

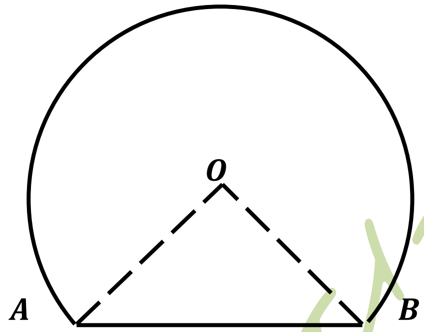
(iv) $f(\theta) = \frac{sin(3\theta)-2,}{tan^2(\theta)}$

(v) $f(\theta) = cos(\theta)sin(\theta) - 1,$

(vi) $f(\theta) = \frac{cos^2(\theta)+1,}{tan\theta}$

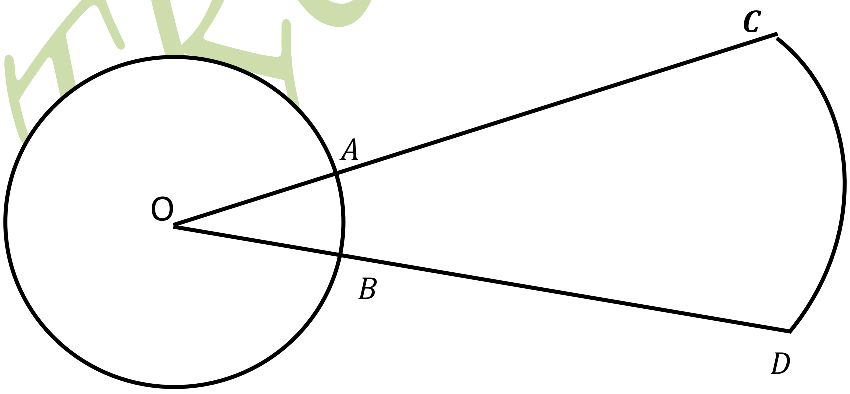
PAST EXAMINATIONS QUESTIONS

ZIMSEC JUNE 2000 PAPER 1



The diagram shows the cross-section of a tunnel. The cross-section has the shape of a major segment of a circle, and the point O is the centre of the circle. The radius of the circle is $4m$, and the size of angle AOB is 1.5 radians. Calculate the perimeter of the cross-section. **[5]**

ZIMSEC JUNE 2006 PAPER 1



The figure shows a circle centre O and radius $2cm$. The radii OA and OB are produced to C and D so that $AC = BD = 10cm$. CD is part of an arc of a circle centre O and radius $12cm$.

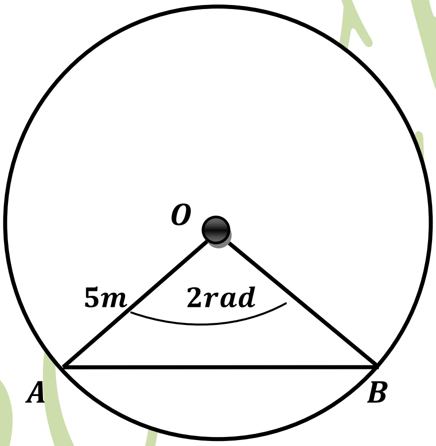
Given that the area of the sector OCD is six times the area of the circle with radius OA, show that the angle $AOB = \frac{\pi}{3} radians$. **[2]**

Hence find

(i) the exact perimeter of the figure $ACDB$ bounded by the straight lines AC, BD and the arcs CD and BA, **[2]**

(ii) the exact length of the line joining B to C. **[2]**

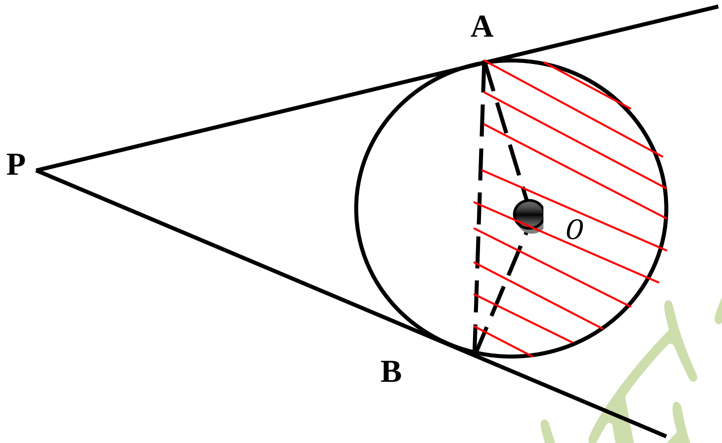
ZIMSEC NOVEMBER 2007 PAPER 1



The diagram shows the cross-section of a circular pond centre O and radius 5 metres. The angle between radii OA and OB is 2 radians. A frog swims in a straight line from A to B and the hops back to A along the minor arc AB. Calculate the total distance travelled by the frog. Calculate the total distance travelled by the frog. **[3]**

ZIMSEC JUNE 2010 PAPER 1

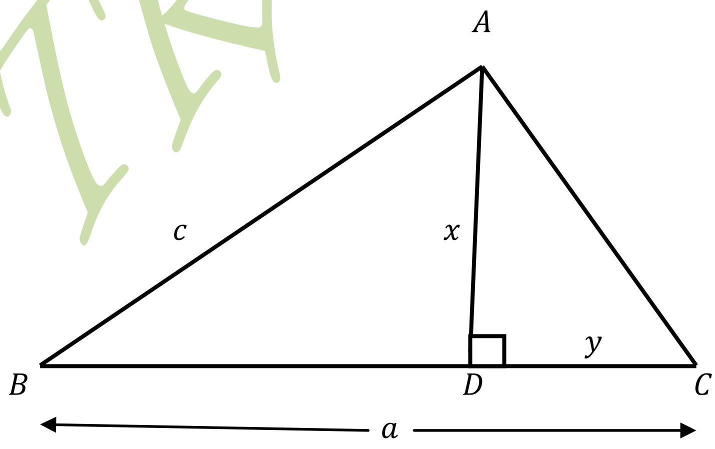
The diagram shows a circle centre O and two tangents AP and BP drawn from a point P.



Given that $AP = 20cm$ and $AB = 12cm$,

(i) Show that the obtuse angle $AOB = 2.532$ radians, **[3]**
(ii) calculate the radius of the circle, **[2]**
(iii) calculate the area of the shaded segment AB. **[3]**

ZIMSEC JUNE 2011 PAPER 1

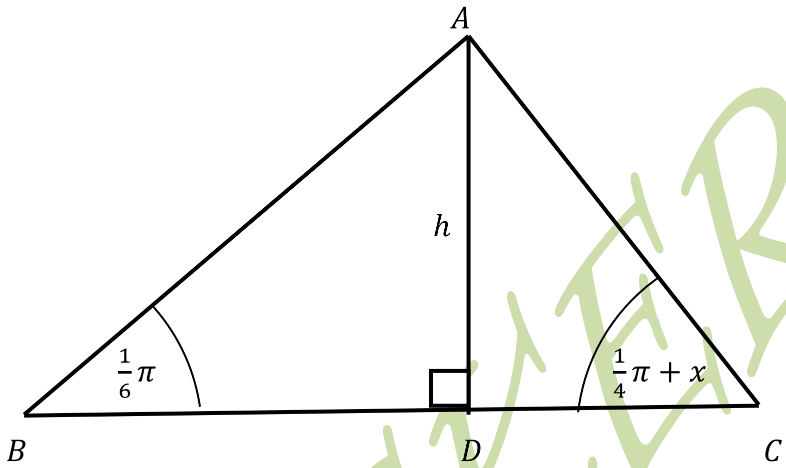


In the diagram above, the lengths of the sides BC, CA and AB are denoted by a, b and c respectively. D is the foor of the perpendicular from A to BC, $AD = x$ and $DC = y$.

(i) Show that $cot\hat{C} = \frac{a}{c}cosec\hat{B} - cot\hat{B}$

(ii) Hence find, correct to the nearest $0.1°$, the angles A and C given that $a = 14{,}7cm, c = 17.3cm$ and $\hat{B} = 64.2°$. **[6]**

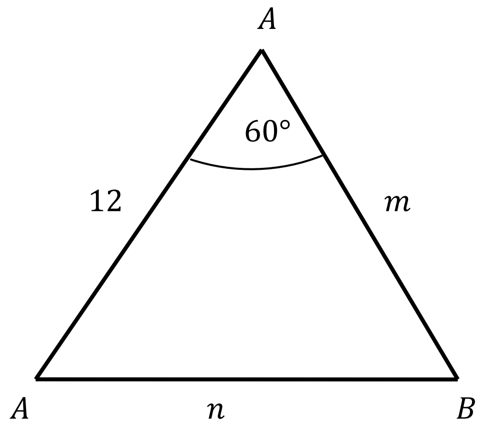
ZIMSEC JUNE 2012 PAPER 1



The diagram shows a triangle ABC with height h units, angle $ABD = \frac{1}{6}\pi$ and angle $ACD = \frac{1}{4}\pi + x$. Given that x is sufficiently small for x^2 and higher powers of x to be neglected, show that

$$BC = h\left[1 + \sqrt{3} - 2x + 2x^2\right].$$ [**5**]

ZIMSEC NOVEMEBER 2012 PAPER 1

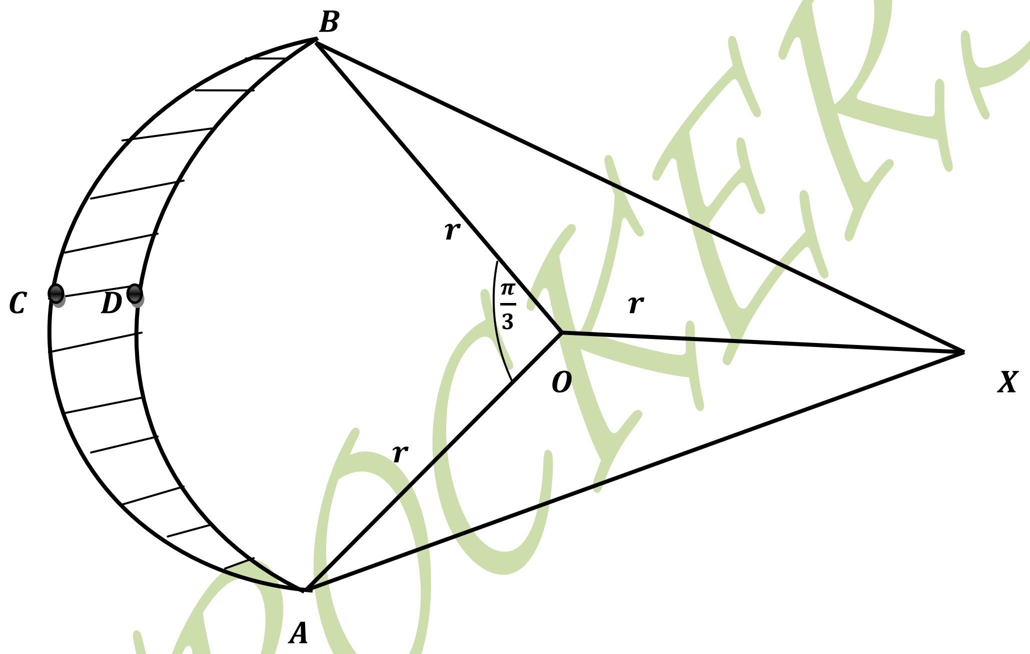


The diagram shows triangle ABC with $AB = m$ cm, $BC = n$ cm, $AC = 12$cm and angle $B\hat{A}C = 60°$.

It is given that the area of triangle ABC is $\sqrt{675}\ cm^2$.

Find the exact values of m and n. **[4]**

ZIMSEC JUNE 2013 PAPER 1



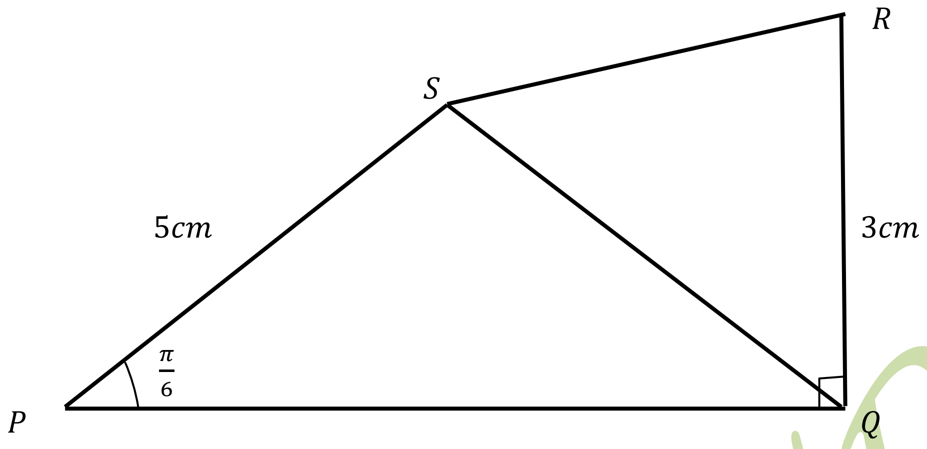
The diagram above shows a school logo. The angle $AOB = \frac{\pi}{3}\ radians$. O is the centre of the sector $OACB$ and X is the centre of Sector $XADB$.

$OX = OA = OB = r\ cm.$

(i) Show that angle $B = \frac{\pi}{6}$. **[2]**

(ii) Calculate the exact area of the shaded region in terms of r. **[6]**

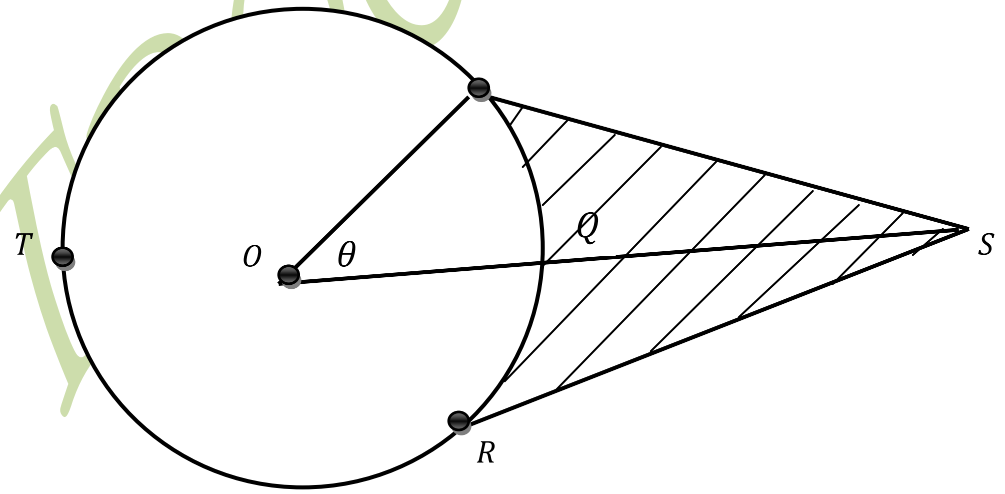
ZIMSEC JUNE 2016 PAPER 1



In the diagram $Q\hat{P}S = \frac{\pi}{6}$, $P\hat{Q}R = \frac{\pi}{2}$, $PS = 5cm$ and $QR = 3cm$.

Given that $n\ P\hat{Q}S = \frac{3}{4}$, find the exact length of RS. **[6]**

ZIMSEC NOVEMBER 2016 PAPER 1

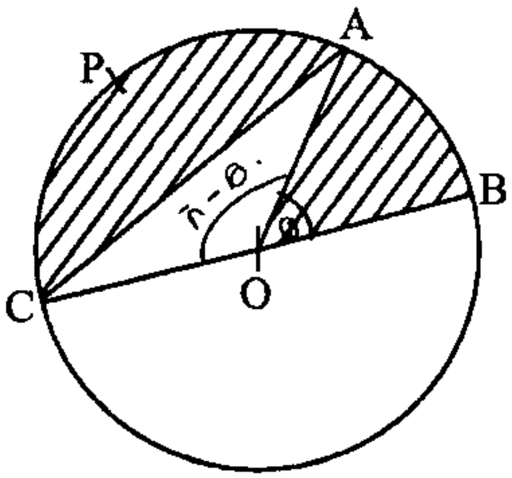


The points P, Q, R and T lie on the circle centre O and radius r. S is on OQ produced such that $QS = r\ cm$ and $PS = RS$. Angle $POS = \theta$ radians.

(i) Calculate the shaded area $PQRS$. **[4]**

(ii) Show that the **perimeter** of $PTRS$ is given by $2r\left(\pi - \theta + 5\sqrt{5 - 4cos\theta}\right)$. **[5]**

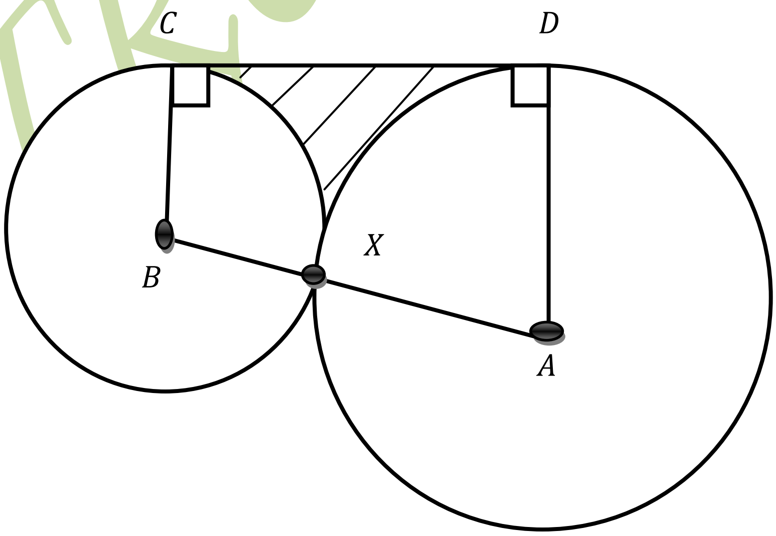
ZIMSEC JUNE 2017 PAPER 1



In the diagram points A, B, C and P are on the circumference of a circle centre O and radius r. BOC is a diameter of the circle. $A\hat{O}B = \theta\ radians.$

Show that the area of the shaded region is $\frac{1}{2}r^2(\pi - sin\theta)$. **[4]**

ZIMSEC NOVEMEBER 2018 PAPER 2



The diagram shows two circles touching at X. The bigger circle has centre A and radius $9cm$. The smaller circle has centre B and radius $3cm$. DC is a common tangent to both circles.

Find the

(i) Perimeter of $ABCD$, **[4]**
(ii) angle BAD, **[2]**
(iii) exact area of the shaded region. **[4]**

ASANTE SANA

*******THERE IS A LIGHT AT THE END OF EVERY TUNNEL *******

CONSTRUCTIVE COMMENTS ON THE FORM OF THE PRESENTATION, INCLUDING ANY OMISSIONS OR ERRORS, ARE WELCOME.

ENJOY

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