Series Expansion

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SYLLABUS (6042) REQUIREMENTS

- Expand expressions of $(a + b)^n$ where n is a positive integer, and of $(1 + x)^n$, where n is a rational number and $|x| < 1$

BINOMIAL THEOREM

- A binomial expression is an algebraic expression which contains two terms

  NB: Bi means two and Nom means term.
- When a binomial expression is raised to any positive power it can be solved by means of binomial theorem

Factorial of a Positive Integer

- The factorial of 'n' denoted by $n!$ is the product of n positive integers from n to 1 (or 1 to n) i.e.

$$n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$$

Example

Write down the value of 5! without using a calculator.

Suggested Solution

$5! = 5 \times 4 \times 3 \times 2 \times 1$

$\quad = 120$

Combination

Definition: Any of the ways we can combine things, when the order does not matter or simply a mathematical technique that determines the number of possible arrangements in a collection of items where the order of the selection does not matter

Example

The combination of $1, 2, 3, 4$ taken three at a time are $321, 142, 413, 234$.

- The numbers of the combination (order does not matter) of n different objects taken 'r' at a time is denoted by $\binom{n}{r}$ or n_{C_r} or $C(n, r)$ and defined as:

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

- The numbers $\binom{n}{r}$ or n_{C_r} or $C(n,r)$ are also called binomial co-efficients

Example

Find the value of $\binom{6}{4}$ without using a calculator.

Suggested Solution

$$\binom{6}{4} = \frac{6!}{4!\,2!}$$

$$= \frac{6\times5\times4\times3\times2\times1}{(4\times3\times2\times1)\times(2\times1)}$$

$$= \frac{6\times5}{2}$$

$$= 15$$

Important Results

1. $\binom{n}{0} = \dfrac{n!}{0!\,(n-0)!} = 1$

2. $\binom{n}{n} = \dfrac{n!}{n!\,(n-n)!} = \dfrac{n!}{n!\times1} = 1$

3. $\binom{n}{r} = \binom{n}{n-r}$

The Binomial Theorem

It is the rule or formula for expansion of $(a+b)^n$, where n is any positive integer and a and b are real numbers.

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + \binom{n}{n}b^n$$

$$\Rightarrow (a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$$

NB: -The coefficients of the successive terms are $\binom{n}{o}$, $\binom{n}{1}$, $\binom{n}{2}$, ... $\binom{n}{r}$ ... $\binom{n}{n}$ and are called Binomial coefficients.

-Sum of binomial coefficients is 2^n

Some important observations in the expansion of $(a+b)^n$

1. The total number of terms is $n+1$, i.e. one more than the exponent n.
2. The 1st term is a^n and $(n+1)^{\text{th}}$ term or the last term is b^n
3. The exponent (power) of 'a' decreases from 'n' to zero.
4. The exponent (power) of 'b' increases from zero to 'n' n.
5. In any term the sum of the indices (exponents) of 'a' and 'b' is equal to n (i.e., the power of the binomial).
6. The coefficients in the expansion follow a certain patterm known as Pascal's triangle.

Another form of the Binomial theorem

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \cdots + b^n$$

Remarks

a) $\binom{n}{1} = \dfrac{n!}{1!\,(n-1)!} = \dfrac{n \times (n-1) \times (n-2) \ldots \times 2 \times 1}{1! \times (n-1) \times (n-2) \times \ldots \times 2 \times 1}$

$= \dfrac{n}{1!}$

$= n$

b) $\binom{n}{2} = \dfrac{n!}{2!\,(n-2)!} = \dfrac{n \times (n-1) \times (n-2) \times (n-3) \times \ldots \times 2 \times 1}{2! \times (n-2) \times (n-3) \times \ldots \times 2 \times 1}$

$$= \frac{n(n-1)}{2!}$$

c) $\binom{n}{3} = \frac{n!}{3!\,(n-3)!} = \frac{n \times (n-1) \times (n-2) \times (n-3) \times \ldots \times 2 \times 1}{3! \times (n-3) \times (n-4)(n-5) \times \ldots \times 2 \times 1}$

$$= \frac{n(n-1)(n-2)}{3!}$$

Worked Problems

Example 1

Expand $(2x-3y)^3$

Suggested Solution

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3$$

Now:

$$(2x-3y)^3 = (2x)^3 + 3(2x)^2(-3y) + \frac{3(2)}{2!}(2x)^1(-3y)^2 + \frac{3(2)(1)}{3!}(2x)^0(-3y)^3$$

$$= 8x^3 - 36x^2y + 54xy^2 - 27y^3$$

Example 2

Expand $(3x+2)^4$

Suggested Solution

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \binom{n}{4}a^{n-4}b^4$$

Now:

$$(3x+2)^4 = \binom{4}{0}(3x)^4 + \binom{4}{1}(3x)^3(2) + \binom{4}{2}(3x)^2(2)^2 + \binom{4}{3}(3x)^1(2)^3 + \binom{4}{4}(3x)^0(2)^4$$

$$= 81x^4 + 216x^3 + 216x^2 + 96x + 16$$

Example 3

Write down the first three terms in the expansion of $(2x^2-2)^8$

Suggested Solution

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots$$

Now:

$$(2x^2-2)^8 = \binom{8}{0}(2x^2)^8 + \binom{8}{1}(2x^2)^7(-2) + \binom{8}{2}(2x^2)^6(-2)^2 + \cdots$$

$$= 256x^{16} - 2048x^{14} + 7168x^{12} + \cdots$$

Example 4

Expand $(p+qx)^4$ up to and including the term in x^2.

Given that the first two terms are $16 - \frac{8}{3}x$, find the values of p and the value of q.

Hence find the third term of the expansion.

Suggested Solution

$(p+qx)^4$

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)a^{n-2}}{2!}b^2 + \cdots$$

Now

$$(p+qx)^4 = p^4 + 4p^3qx + 6p^2q^2x^2.$$

Given $p^4 + 4p^3qx \equiv 16 - \frac{8}{3}x$

$\Longrightarrow p^4 = 16$

$\therefore p = 2$ and

$4p^3q = -\frac{8}{3}$

$$\Rightarrow q = -\frac{8}{3(32)}$$

$$\therefore q = -\frac{1}{12}$$

Finding the third term of expansion:

$$6p^2q^2x^2 = 6(2^2)\left(-\frac{1}{12}\right)^2 x^2$$

$$= \frac{6 \times 4}{144} x^2$$

$\therefore$ The third term is $\frac{1}{6}x^2$

The Pascal's Triangle

Definition: A triangular array of numbers in which those at the ends of the rows are 1 and each of the others is the sum of the nearest two numbers in the row above e.g. from the pattern below: $1 + 4 = 5$ (highlighted in red) and $1 + 2 = 3$ (highlighted in blue)

Index of Binomial	Coefficient of various terms
0	1
1	1 1
2	1 **2** **1**
3	1 3 **3** 1
4	1 4 6 4 1
5	1 **5** 10 10 5 1

Observations from the Pascal's triangle

1. Each number is the sum of the two numbers diagonally above it (with the exception of the $1's$).
2. Each row is symmetric (i.e., the same backwards as forwards).
3. The sum of the numbers in each row is a power of 2.

Worked Problems

Example 1

Expand $(2x - 3y)^3$

Suggested Solution

The exponent is 3 thus we use the patterm 1 3 3 1

Now:

$$(2x - 3y)^3 = \mathbf{1}(2x)^3(-3y)^{\mathbf{0}} + \mathbf{3}(2x)^2(-3y)^{\mathbf{1}} + \mathbf{3}(2x)^1(-3y)^{\mathbf{2}} + \mathbf{1}(2x)^0(-3y)^{\mathbf{3}}$$

$$= 8x^3 - 36x^2y + 54xy^2 - 27y^3$$

Example 2

Expand $(3x + 2)^4$

Suggested Solution

The exponent is 4 thus we use the patterm 1 4 6 4 1

Now:

$$(3x + 2)^4 = \mathbf{1}(3x)^4(2)^{\mathbf{0}} + \mathbf{4}(3x)^3(2)^{\mathbf{1}} + \mathbf{6}(3x)^2(2)^{\mathbf{2}} + \mathbf{4}(3x)^1(2)^{\mathbf{3}} + \mathbf{1}(3x)^0(2)^{\mathbf{4}}$$

$$= 81x^4 + 216x^3 + 216x^2 + 96x + 16$$

The General Term

- It is used to find out the specified term or the required co-efficient of the term in the binomial expansion
- The term $\binom{n}{r} a^{n-r} b^r$ in the expansion of binomial theorem is called the General term or $(r+1)^{\text{th}}$ term.
- It is denoted by T_{r+1}.
- Hence

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

Worked Examples

Example 1

Find the coefficient of x^{-2} in the expansion of $\left(x+\frac{2}{x^2}\right)^7$.

Suggested Solution

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r$$

$$\Rightarrow T_{r+1}=\binom{7}{r}x^{7-r}\left(\frac{2}{x^2}\right)^r$$

Aside

Finding r:

$$\frac{x^{7-r}}{x^{2r}}\equiv x^{-2}$$

$$\Rightarrow x^{7-3r}\equiv x^{-2}$$

$$\Rightarrow 7-3r=-2$$

$$\Rightarrow\therefore r=3$$

Now:

$$\Rightarrow T_4=\binom{7}{4}x^{7-3}\left(\frac{2}{x^2}\right)^3$$

$$\Rightarrow T_4=\binom{7}{4}x^4\frac{8}{x^6}$$

$$\Rightarrow T_4=\binom{7}{4}\times 8x^{-2}$$

$$\Rightarrow T_4=\binom{7}{4}\times 8x^{-2}$$

$$\Rightarrow T_4=280x^{-2}$$

$\therefore$The coefficient of x^{-2} is 280.

Example 2

Find the constant term in the expansion of $\left(x^2+\frac{1}{x}\right)^9$.

<u>Suggested Solution</u>

General term $T_{r+1}=\binom{n}{r}a^{n-r}b^r$: $a=x^2$, $b=\frac{1}{x}$ and $n=9$.

Now:

$$T_{r+1}=\binom{9}{r}(x^2)^{9-r}\left(\frac{1}{x}\right)^r$$

<u>Aside</u>

Finding r:

$$(x^2)^{9-r}\left(\frac{1}{x}\right)^r\equiv x^0$$

$$\Rightarrow \frac{x^{18-2r}}{x^r}\equiv x^0$$

$$\Rightarrow x^{18-3r}\equiv x^0$$

$$\Rightarrow 18-3r=0$$

$$\Rightarrow r=6$$

Now:

$$T_7=\binom{9}{6}(x^2)^{9-6}\left(\frac{1}{x}\right)^6$$

$$\Rightarrow T_7=\binom{9}{6}(x^2)^3\left(\frac{1}{x}\right)^6$$

$$\Rightarrow T_7=\binom{9}{6}x^6\left(\frac{1}{x}\right)^6$$

$$\Rightarrow T_7=\binom{9}{6}$$

$$\Rightarrow T_7=84$$

$\therefore$ The constant term is 84.

Example 3

Find the term which is independent of x in the expansion of $\left(\frac{1}{2x} - \frac{x^2}{3}\right)^9$.

Suggested Solution

$$\left(\frac{1}{2x} - \frac{x^2}{3}\right)^9$$

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$\Rightarrow T_{r+1} = \binom{9}{r} \left(\frac{1}{2x}\right)^{9-r} \left(\frac{-x^2}{3}\right)^r$$

Aside

Finding r:

$$\Rightarrow \frac{(x^2)^r}{x^{9-r}} \equiv x^0$$

$$\Rightarrow x^{2r+r-9} \equiv x^0$$

$$\Rightarrow 3x = 9$$

$$\therefore r = 3.$$

Now

$$T_4 = \binom{9}{3} \left(\frac{1}{2x}\right)^6 \left(\frac{-x^2}{3}\right)^3.$$

$$= -\frac{7}{144}$$

Binomial Series

- The Binomial Series is used if n is a negative integer or is a fractional number (either negative or positive)

- This series is valid only when x is numerically less than unity i.e. $|x| < 1$, otherwise the expression will not be valid.
- It is the Maclaurin series expansion of the function $(1+x)^n$ where:

$$(1+x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{r}x^r + \cdots + \binom{n}{n}x^n$$

or

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + x^n$$

NB: The Binomial formula of $(a+b)^n$ is valid when 'n' is a positive integer but if 'n' is not a positive integer (negative or fraction) to expand $(a+b)^n$, we write it as:

$$(a+b)^n = a^n\left(1+\frac{b}{a}\right)^n$$

where $\left|\frac{b}{a}\right|$ must be less than 1 i.e. $\left|\frac{b}{a}\right| < 1$.

Application of the Binomial Series

Approximations

The binomial series can be used to find expression approximately equal to the given expressions under given conditions.

Worked Examples

Example 1

If x is very small, so that its square and higher powers can be neglected then show that

$$\frac{x}{(1-x)^3} \approx x + 3x^2$$

Suggested Solution

$$\frac{x}{(1-x)^3} \equiv x(1-x)^{-3}$$

$$(1+x)^n = 1 + nx + \cdots$$

Now

$$x(1-x)^{-3} = x[1 + (-3)(-x) + \cdots]$$

$$= x(1 + 3x + \cdots)$$

$$\approx x + 3x^2 \quad (as\ required)$$

Example 2

a) Given that

$$f(x) = \frac{x^2 + 8x + 13}{(x+3)(x+2)},$$

express $f(x)$ in partial fractions.

b) Hence find the values of a, b and c if $f(x) \equiv a + bx + cx^2 + \cdots$

<u>Suggested Solution</u>

a) $f(x) = \frac{x^2+8x+13}{(x+3)(x+2)}$

$$f(x) = \frac{x^2+8x+13}{x^2+5x+6} \equiv \frac{x^2+5x+3x+6+7}{x^2+5x+6} \equiv \frac{x^2+5x+6}{x^2+5x+6} + \frac{3x+7}{x^2+5x+6}$$

$$\equiv 1 + \frac{3x+7}{x^2+5x+6}$$

Let $p(x) = \frac{3x+7}{(x+3)(x+2)} \equiv \frac{A}{(x+3)} + \frac{B}{(x+2)}$

$$\equiv \frac{A(x+2) + B(x+3)}{(x+3)(x+2)}$$

$\Rightarrow A(x+2) + B(x+3) = 3x + 7$

Now when $x = -2$:

$B = 1$

when $x = -3$

$-A = -2$

$\therefore A = 2$

$$\therefore f(x) = 1 + \frac{2}{x+3} + \frac{1}{x+2}$$

b) $f(x) \equiv a + bx + cx^2 + \cdots$

$$f(x) = 1 + \frac{2}{x+3} + \frac{1}{x+2}$$

$$= 1 + 2(x+3)^{-1} + (x+2)^{-1}$$

$$= 1 + 2(3)^{-1}\left(1 + \frac{x}{3}\right)^{-1} + (2)^{-1}\left(1 + \frac{x}{2}\right)^{-1}$$

Now:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots$$

$$\Rightarrow f(x) = 1 + \frac{2}{3}\left[1 + (-1)\left(\frac{x}{3}\right) + \frac{(-1)(-2)}{2 \times 1}\left(\frac{x}{3}\right)^2\right] + \frac{1}{2}\left[1 + (-1)\left(\frac{x}{2}\right) + \frac{(-1)(-2)}{2 \times 1}\left(\frac{x}{2}\right)^2\right]$$

$$= 1 + \frac{2}{3}\left[1 - \frac{1}{3}x + \frac{1}{9}x^2\right] + \frac{1}{2}\left[1 - \frac{1}{2}x + \frac{1}{4}x^2\right]$$

$$= 1 + \frac{2}{3} - \frac{2}{9}x + \frac{2}{27}x^2 + \frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x^2 + \cdots$$

$$= 1 + \frac{2}{3} + \frac{1}{2} - \frac{2}{9}x - \frac{1}{4}x + \frac{2}{27}x^2 + \frac{1}{8}x^2 + \cdots$$

$$= \frac{13}{6} - \frac{17}{36}x + \frac{43}{216}x^2 + \cdots$$

$\therefore a = \frac{13}{6}$; $\quad b = -\frac{17}{36}$ and $\quad c = \frac{43}{216}$

Root Extraction

The second application of the binomial series is that of finding the root of any quantity.

Solved Problems

Example 1

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(i) Expand $(9-4x)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

(ii) By using $x=\frac{1}{9}$ in the expansion, find an approximation for $\sqrt{77}$, leaving your answer in the form $\frac{p}{q}$.

Suggested Solution

(i) $(9-4x)^{-\left(\frac{1}{2}\right)} = 9^{-\frac{1}{2}}\left(1-\frac{4x}{9}\right)^{-\left(\frac{1}{2}\right)}$

$$=\frac{1}{3}\left(1-\frac{4x}{9}\right)^{-\left(\frac{1}{2}\right)}$$

Using $(1+x)^{n} = 1+nx+\frac{n(n-1)}{2!}x^{2}+\cdots$

$$\frac{1}{3}\left(1-\frac{4x}{9}\right)^{-\left(\frac{1}{2}\right)} = \frac{1}{3}+\frac{2}{27}x+\frac{2}{81}x^{2}$$

Finding range of values of x for which the above expansion is valid.

$$\left|-\frac{4x}{9}\right|<1 \Rightarrow \quad |x|<\frac{9}{4}$$

(ii) $(9-4x)^{-\left(\frac{1}{2}\right)} = \frac{1}{3}+\frac{2}{27}x+\frac{2}{81}x^{2}$

Substituting $x=\frac{1}{9}$ both sides:

$$\left[9-4\left(\frac{1}{9}\right)\right]^{-\left(\frac{1}{2}\right)} = \frac{1}{3}+\frac{2}{27}\left(\frac{1}{9}\right)+\frac{2}{81}\left(\frac{1}{9}\right)^{2}$$

Simplify the coefficients to get:

$$\frac{3}{\sqrt{77}} = \frac{2243}{6561}$$

$$\therefore \sqrt{77} = \frac{19\,683}{2243}$$

Example 2

(i) Find the first 4 terms in the expansion of $\sqrt[3]{(1-2x)}$.

(ii) Hence

1. State the range of values for which the expansion is valid,
2. By putting $x = \frac{1}{27}$, show that $\sqrt[3]{25} \approx 2.92402$ correct to5 decimal places.

Suggested Solution

(i) $\sqrt[3]{(1-2x)} \equiv (1-2x)^{\frac{1}{3}}$

Now:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3$$

$$\Rightarrow (1-2x)^{\frac{1}{3}} = 1 + \left(\frac{1}{3}\right)(-2x) + \frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)}{2\times 1}(-2x)^2 + \frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3\times 2\times 1}(-2x)^3$$

$$= 1 - \frac{2}{3}x + \left(\frac{1}{3}\right)\left(-\frac{1}{3}\right)\times 4x^2 + \left(\frac{1}{9}\right)\left(-\frac{1}{3}\right)\left(-\frac{5}{3}\right)\times -8x^3$$

$$= 1 - \frac{2}{3}x - \frac{4}{9}x^2 - \frac{40}{81}x^3$$

(ii) Hence means use the above result i.e.

1. $|-2x| < 1$

$|2x| < 1$

$|x| < \frac{1}{2}$ or $-\frac{1}{2} < x < \frac{1}{2}$

$$2.\ (1-2x)^{\frac{1}{3}} = 1 - \frac{2}{3}x - \frac{4}{9}x^2 - \frac{40}{81}x^3$$

$$\left[1-2\left(\frac{1}{27}\right)\right]^{\frac{1}{3}} = 1 - \frac{2}{3}\left(\frac{1}{27}\right) - \frac{4}{9}\left(\frac{1}{27}\right)^2 - \frac{40}{81}\left(\frac{1}{27}\right)^3$$

$$\left(1-\frac{2}{27}\right)^{\frac{1}{3}} = 1 - \frac{2}{81} - \frac{4}{9}\left(\frac{1}{729}\right) - \frac{40}{81}\left(\frac{1}{19683}\right)$$

$$\left(\frac{25}{27}\right)^{\frac{1}{3}} = 1 - \frac{2}{81} - \frac{4}{6561} - \frac{40}{1594323}$$

$$\frac{(25)^{\frac{1}{3}}}{3} = 0.974673889$$

$$(25)^{\frac{1}{3}} = 3(0.974673889)$$

$$\sqrt[3]{25} = 2.924021669$$

$\therefore\ \sqrt[3]{25} \approx 2.92402$ to 5 decimal places (as required)

Example 3

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It is given that $f(x) = \sqrt{\frac{1-x}{1+x}}$

(i) If x is small enough for its cube and higher powers to be neglected, show that

$$f(x) = 1 - x + \frac{x^2}{2}.$$

(ii) By putting $x = \frac{1}{8}$, show that $\sqrt{7} \approx 2\frac{83}{128}$.

Suggested Solution

(i) $f(x) = \sqrt{\dfrac{1-x}{1+x}}$

Now

$$\sqrt{\frac{1-x}{1+x}} = \left(\frac{1-x}{1+x}\right)^{\frac{1}{2}} \equiv (1-x)^{\frac{1}{2}}(1+x)^{-\frac{1}{2}}$$

Now using

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2$$

Aside:

$$(1-x)^{\frac{1}{2}} = 1 + \frac{1}{2}(-x) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}(-x)^2$$

$$\approx 1 - \frac{1}{2}x - \frac{1}{8}x^2 \qquad \text{(i)}$$

$$(1+x)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)x + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}x^2$$

$$\approx 1 - \frac{1}{2}x + \frac{3}{8}x^2 \qquad \text{(ii)}$$

Multiplying equations (i) and (ii) together:

$$f(x) = \left(1 - \tfrac{1}{2}x - \tfrac{1}{8}x^2\right)\left(1 - \tfrac{1}{2}x + \tfrac{3}{8}x^2\right)$$

$$= 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{1}{2}x + \frac{1}{4}x^2 - \frac{1}{8}x^2$$

$$= 1 - x + \frac{1}{2}x^2.$$

(ii) $\sqrt{\dfrac{1-x}{1+x}} = 1 - x + \dfrac{x^2}{2}$

Putting $x = \frac{1}{8}$ on both sides:

$$\sqrt{\frac{1-\frac{1}{8}}{1+\frac{1}{8}}} = 1 - \frac{1}{8} + \frac{\left(\frac{1}{8}\right)^2}{2}$$

$$\frac{\sqrt{7}}{3} = \frac{113}{128}$$

$$\Rightarrow \sqrt{7} \approx 2\tfrac{83}{128}.$$

PAST EXAMINATION QUESTIONS

UCLES NOVEMBER 1996 PAPER 1

Expand $(1+x)^{-4}$, where $|x|<1$, as a series of ascending powers of x, up to and including the term in x^3, simplifying the coefficients. **[4]**

UCLES JUNE 1998 PAPER 1

Find the series expansion of $(1+2x)^{\frac{5}{2}}$, up to and including the term in x^3, simplifying the coefficients. **[3]**

UCLES NOVEMBER 1998 PAPER 1

Expand $(1-2x)^{-\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^2. **[3]**

State the set of values for which the expansion is valid. **[1]**

Calculate the relative error in using these terms of the expansion as an approximation for $(1-2x)^{-\frac{1}{2}}$ when $x=0.1$. **[2]**

UCLES JUNE 2000 PAPER 1

Write down the expansion $(1+x)^5$. **[1]**

Hence, by letting $x=z+z^2$, find the coefficient of z^3 in the expansion of $(1+z+z^2)^5$ in the powers of z. **[4]**

UCLES NOVEMBER 2000 PAPER 1

Find the term which is independent of x in the expansion of

$$\left(x - \frac{2}{x^2}\right)^6$$ [3]

UCLES NOVEMBER 2000 PAPER 1

Expand $(1 - 4x)^{-3}$ in ascending powers of x, up to and including the term in x^2. simplifying the coefficients. **[3]**

ZIMSEC NOVEMBER 2003 PAPER 1

Expand $(8 + x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^3,simplifying the coefficients.

State the set of values for which the expansion is valid. **[5]**

ZIMSEC JUNE 2004 PAPER 1

Find the coefficient of x^6 in the expansion of $(9 + 3x^2)^{\frac{1}{2}}$ in ascending powers of x. **[3]**

ZIMSEC NOVEMBER 2004 PAPER 1

(i) Expand $(1 - x)^{-\frac{1}{2}}$ up to and including the term in x^3, simplifying your coefficients. **[3]**

(ii) Hence by putting $x = \frac{1}{50}$, estimate $\sqrt{2}$ correct to six decimal places. **[4]**

ZIMSEC NOVJUNE 2005 PAPER 1

Expand $(1 + 2x)^4$ in ascending powers of x. **[2]**

Hence find the coefficient of the term in x^3 of the expansion of

$(2x+3)(1+2x)^4$. **[2]**

ZIMSEC NOVEMBER 2006 PAPER 1

Obtain the first four terms of the expansion of $(1-3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^2. **[3]**

State the set of values of x for which the expansion is valid. **[1]**

By putting $x = 0.0002$ in your expansion, find $\frac{1}{\sqrt[3]{0.9994}}$ correct to 11 decimal places. **[2]**

ZIMSEC JUNE 2007 PAPER 1

Find the coefficient of x^6 in the expansion of $(9+x^2)^{\frac{1}{2}}$ in ascending powers of x. **[3]**

ZIMSEC NOVEMBER 2009 PAPER 1

Find the coefficient of x^{-3} in the expansion of

$$\left(x^2 - \frac{2}{x}\right)^9$$ [**4**]

ZIMSEC JUNE 2010 PAPER 1

(a) Find the term independent of x in the expansion of

$$\left(x^2 + \frac{3}{x}\right)^6$$ [**2**]

(b) Find the series expansion of $(4+x^2)^{\frac{1}{2}}$ up to and including the term in x^6. **[4]**

ZIMSEC JUNE 2011 PAPER 1

Find the series expansion of $\sqrt{(4-3x^2)}$ up to and including the term in x^4. **[4]**

ZIMSEC NOVEMBER 2011 PAPER 1

It is given that $f(x) = \frac{1}{(1+x)^2} + \sqrt{9+x}$

(a) Expand $f(x)$ up to and including the term in x^2 simplifying coefficients. **[7]**

(b) State the set of values of x for which the expansion is valid. **[1]**

ZIMSEC JUNE 2012 PAPER 1

If the expansion of $(1+x)^{\frac{1}{5}}$ and $\frac{1+px}{1+qx}$ in ascending powers of x are identical up to and including the term in x^2, calculate the value of p and the value of q. **[7]**

ZIMSEC JUNE 2013 PAPER 1

(i) Express $\frac{4x+6}{(x+2)(x+1)(x+3)}$ in partial fractions. **[3]**

(ii) By using a series expansion up to and including the term in x^2 show that $\frac{4x+6}{(x+2)(x+1)(x+3)}$ can be reduced to $1 - \frac{7}{6}x + \frac{41}{36}x^2$. **[4]**

ZIMSEC NOVEMBER 2013 PAPER 1

Find the series expansion of $\frac{(4-x)^{\frac{1}{2}}}{2x^2-1}$ up to and including the term in x^2.

State the set of values of x for which the expansion is valid. **[5]**

ZIMSEC JUNE 2014 PAPER 1

Find the coefficient of x^2 in the expansion of

$$\left(x - \frac{5}{x}\right)^6$$ [3]

ZIMSEC NOVEMBER 2014 PAPER 1

(i) Use binomial expansion to simplify $(x + 2)^7 - (x - 2)^7$. **[6]**

(ii) Hence use your answer in (**i**) to find the exact value of

$\left(\sqrt{5} + 2\right)^7 - \left(\sqrt{5} - 2\right)^7$. **[2]**

ZIMSEC JUNE 2015 PAPER 1

(i) The expansion of $(1 + ax)^n$ up to and including the term in x^2 is $1 - 6x + \frac{81}{4}x^2$.

Find the values of a and n. **[5]**

(ii) Hence state the set of values of x for which the expansion is valid. **[1]**

ZIMSEC NOVEMBER 2015 PAPER 1

(i) Expand $(3 - 2x)^{-3}$ up to and including the term in x^3, simplifying the coefficients.

(ii) Hence state the set of values of x for which the expansion in (**i**) is valid. **[5]**

ZIMSEC NOVEMBER 2016 PAPER 1

Find the series expansion of $(9 + 2x)^{-\frac{3}{2}}$ up to and including the term in x^2, simplifying the coefficients. **[4]**

ZIMSEC JUNE 2017 PAPER 1

(a) Expand $\frac{1}{\sqrt{1-2x}}$ up to and including the term in x^2. **[3]**

(b) State the set of values of x for which the expansion is valid. **[1]**

(c) Taking $= -\frac{1}{6}$, use the expansion to find $\sqrt{3}$, correct to 5 decimal places. **[2]**

ZIMSEC JUNE 2018 PAPER 1

Find the term independent of x in the expansion of

$$\left(\frac{1}{x^2} - \frac{2x}{3}\right)^6.$$ [**4**]

ZIMSEC NOVEMBER 2018 PAPER 1

The expansion of $(1 + ax)^n$, where $n < 0$, has the coefficient of the terms in x and x^2 equal to -1 and $\frac{81}{4}$ respectively.

(a) Show that n satfies the equation

$27n^2 + 3n - 2 = 0.$ **[4]**

(b) (i) Find the value of n

(ii) Find the value of a.

(iii) Hence, state the set of values of x for which the expansion is valid. **[5]**

ZIMSEC NOVEMBER 2019 PAPER 1

Find the term independent of y in the expansion of

$$\left(y^2 - \frac{4}{y}\right)^6.$$ [

ASANTE SANA

********THERE IS A LIGHT AT THE END OF EVERY TUNNEL ********

CONSTRUCTIVE COMMENTS ON THE FORM OF THE PRESENTATION, INCLUDING ANY OMISSIONS OR ERRORS, ARE WELCOME.

ENJOY

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